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**Visualizing Qubit Entanglement**

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*Abstract*—First, we show a general method of visualizing n-dimensional “cubes” for n=1, 2, 3, 4, and higher. Second, we imbed the 2-D single qubit superposition states in the computational basis (|0> and |1>) in the 4-dimensional picture. Third, we choose a set of basis vectors for the full 4-dimensional space which is the tensor product space of 2 single qubits (in the computational basis). Fourth, using the choice of a 4-D basis, we map the first two (entangled) Bell States |+> and |->) to a 2-D Plane in the 4-D space. Fifth, using the same choice of a 4-D basis, we map the second two (entangled) Bell States |+> and |->) to a 2-D Plane in the 4-D space. Sixth we show the Bell states span the 4-D space (are a basis) and are pairwise independent of each other. In this basis it is clear that the superposition states are independent of all of the Bell states. It is also geometrically clear that most (virtually ALL) states are entangled (qubit states are interior and surface points of the 4 D hypercube).

*Index Terms*—Entangled, Qubit, State, Visualization.

# INTRODUCTION

First, we show a general method of visualizing n-dimensional “cubes” for n=1, 2, 3, 4, and higher. Second, we imbed the 2-D single qubit superposition states in the computational basis (|0> and |1>) in the 4-dimensional picture. Third, we choose a set of basis vectors for the full 4-dimensional space which is the tensor product space of 2 single qubits (in the computational basis). Fourth, using the choice of a 4-D basis, we map the first two (entangled) Bell States |+> and |->) to a 2-D Plane in the 4-D space. Fifth, using the same choice of a 4-D basis, we map the second two (entangled) Bell States |+> and |->) to a 2-D Plane in the 4-D space. Sixth we show the Bell states span the 4-D space (are a basis) and are pairwise independent of each other. In this basis it is clear that the superposition states are independent of all of the Bell states. It is also geometrically clear that most (virtually ALL) states are entangled (qubit states are interior and surface points of the 4 D hypercube).

# a general method of visualizing n-dimensional “cubes**”**

In order to visualize the single and double qubit Hilbert states of entanglement, we build up a more easily visualized 4-D hyper “cube” (or brick) as a visualization aid. Then the infinite 4-D planes ofthe Hilbert space become more easily manipulated & visualized.

If the edge lengths were in fact all equal, we would have a true hypercube. The length of the edges is not an issue for us since we reallyare interested in the infinite extent, both plus and minus of all edges.

## The Algorithm for Building Hyper Cubes



## Basic Cases





## The Sub Array Counts for the Case n=4.

The counts for the 4-D Hyper “Cube” are

1. n=4
* The dimension
* The incidence number of edges on each node
1. The number of nodes =16
* 8 for the “base” cube
* 8 for the “top” cube
1. The number of edges = 32
* 12 in the base
* 12 in the top
* 8 connecting the “bottom” nodes to the corresponding “top” nodes
1. There are 24 2-D faces.
* 6 in the base
* 6 in the top
* For each pair of adjacent nodes (edges) in the base, there is a 4-D face ending on the corresponding nodes in the top cube. There are 12 edges in the base cube.
1. There are 8 3-D cubes.
* The base
* The top
* For each of the 6 faces of the bottom cube, there is a cube built on it whose top is the corresponding face on the top cube.

## Basis Vectors and 3-D Cubes.

Base & “Top” Cube (2)





Toward & Away Faces Cubes (2) & Top & Bottom Faces Cubes (2)





Right Facing Cube & Left Facing Cube (2)





# Tensor Products of Qubits and Basis Vectors

## Tensor Products

|0> is the computational basis vector of a single qubit.

|1> is the other computational basis vector of a single qubit.

They are orthogonal and form a basis for single qubit states.

 (1)

The tensor product of 2, 2-D qubits is a 4-D vector.

 (2)

 (3)

 (4)

 (5)

(2), (3), (4), and (5) are 2-qubit state vectors that form an orthonormal basis set for the 2-qubit 4-D tensor product space.

Using (1), the general 2-qubit state is:

 (6)

## 2-D & 4-D Basis Vectors

Consider a 2x2 array A. It has 4 “cells.” The “contents” of the cells are determined by pointers to a codomain of Contents.

Now consider a 2x2x1 array B based on A. It too has only 4 cells whose contents are determined the same way, by pointers to a codomain of Contents.

The continuous cases (x, y) and (x, y, z) work the same way only now the cells are associated with points as are the pointers.

Both arrays, a 2-D A and a 3-D B have the same ability to store 4 contents. The 2-D pointers can differ from the 3-D pointers, so the contents for the same geographical points can differ based on their dimension.

The lesson learned is: if a 2-D area is shared as the base of a 3-D object, the points of both have separate associated pointers to contents which can differ based on their dimension.

## We Associate the 2-D Basis with the 4-D Basis.

The 2- D basis:  (7)

The 4- D basis:

  (8)

 (9)

# The Locations of the 2-D & 4-D Basis Vectors for the 4-D Hyper “Cube”



The |0>|0> - |1>|1> Plane lies outside of the base cube.

## Where are the |0> & |1> superposition states?

 (10)



*They lie in the |0>, |1> unbounded plane*. They are vectors lying in the plane determined by |0> and |1>. It is the plane of the base of the bottom cube. Because (2+2=1), they lie on the unit circle about the origin. All other dimensions are null.

## Where are the Bell States?

 (11)

Where are the Bell States? They lie in the |00>, |11> unbounded plane and the |01>, |10> unbounded plane. The |00>, |11> plane is a plane between the bottom and top of the hypercube.

The |01>, |10> plane is the vertical face plane of the bottom cube.

It is geometrically clear that the Bell states can’t be built up out of the computational basis states since they lie in essentially independent planes from the plane of the computational basis. Also their two independent planes form a basis of the 4-D space.

 (12)

It is also geometrically clear that since the superposition states are the linear composition of the computational basis and lie in the base plane of the bottom cube, **almost all other states (points in and on the hypercube), are entangled states**.

Proof that the Bell states are not a tensor product of single qubit computational basis states.



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