Bayesian Analysis of the Systemic Risk Ratings using Generalized Threshold GARCH Model

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Introduction

I propose Bayesian Markov Chain Monte Carlo (MCMC) estimation of the systemic risk rankings based on the MES (marginal expected shortfall) and SRISK (the expected capital shortage of a firm conditional on a substantial market decline). I also introduce a generalized threshold conditional volatility model (GTARCH) and compare it to traditional asymmetric models of volatility. The new model allows both ARCH and GARCH parameters to change when previous period return is negative, eliminates the problem of a negative bias of alpha in the traditional GJR-GARCH model and shows higher volatility persistence for negative returns compared to GJR-GARCH. I apply the GTARCH model for forecasting volatility of financial institutions equity returns. The equity volatility combined with correlation with the market are used for the measurement of systemic risks, MES and SRISK, in a fashion similar to Brownlees and Engle (2012) and Acharya et. al (2010) but incorporating better asymmetric volatility properties and uncertainty for risk measures.

Measurement of Systemic Risk

Let r_t and $r_{m,t}$ be the daily log returns of a firm and the market correspondingly. Following Brownlees and Engle (2012) we consider the following model for the returns:

$$\begin{aligned} r_{mt} &= \sigma_{mt}\epsilon_{mt} \\ r_t &= \sigma_t \rho_t \epsilon_{mt} + \sigma_t \sqrt{1-\rho_t^2}\epsilon_t \end{aligned} \qquad (1)$$

where $\epsilon_{mt}, \epsilon_t \sim F$ are independent and identically distributed variables with zero means and unit variances, σ_t and σ_{mt} are conditional standard deviations of the firm return and the market return correspondingly, and ρ_t is conditional correlation between the firm and the market. This model is also called the dynamic conditional beta model with $\beta_t = \rho_t \dfrac{\sigma_t}{\sigma_{mt}}$ and tail dependence on correlation of firm returns and the market

Measurement of Systemic Risk

The first considered systemic risk measure is the daily marginal expected shortfall (MES) which is the conditional expectation of a daily return of a financial institution given that the market return falls below threshold level C. In practice, $C = -2\%$.

$$
\begin{aligned}
MES_{t-1} &= E_{t-1}(r_t | r_{mt} < C) \\
&= \sigma_t \rho_t E_{t-1}(\epsilon_{mt} | \epsilon_{mt} \leq C/\sigma_{mt}) + \sigma_t \sqrt{1-\rho_t^2} E_{t-1}(\epsilon_t | \epsilon_{mt} \leq C/\sigma_{mt})
\end{aligned} \quad (3)
$$

The computation of the expected shortfall following Scaillet (2005) using nonparametric estimates given by:

$$
E_{t-1}(e_{mt} | e_{mt} \leq \alpha) = \frac{\sum_{i=1}^{t-1} e_{mi} \Phi_h(\frac{\alpha - e_{mi}}{h})}{\sum_{i=1}^{t-1} \Phi_h(\frac{\alpha - e_{mi}}{h})} \quad (4)
$$

where $\alpha = C/\sigma_{mt}$, $\Phi_h(t) = \int_{-\infty}^{t/h} \phi(u) du$, $\phi(u)$ is a standard normal probability distribution function used as kernel, and $h = T^{-1/5}$ is the bandwidth parameter.

Measurement of Systemic Risk

The second measure is the long run marginal expected shortfall based on the expectation of the cumulative six month firm return conditioned on the event that the market falls by more than 40% in six months.

It is shown by Acharya, Engle and Richardson (2012) that the LRMES can be approximated by

$$LRMES_t \approx 1 - e^{-18*MES_t} \tag{5}$$

Finally, the capital shortfall of the firm based on the potential capital loss in six months is defined as

$$SRISK_t = max\{0; kD_t - (1-k)(1 - LRMES_t)E_t\} \tag{6}$$

where D_t is the book value of Debt at time t, E_t is the market value of equity at time t and $k \approx 8\%$ is the prudential capital ratio of the US banks. It is assumed that the capital loss happens only due to the loss in the market capitalization $LRMES * E_t$

Generalized Threshold GARCH model

Threshold ARCH or GJR-GARCH(1,1,1)

$$
\begin{aligned}
y_t &= \mu + \epsilon_t \\
\sigma_t^2 &= \omega + \alpha\epsilon_{t-1}^2 + \gamma\epsilon_{t-1}^2 I(y_{t-1} - \mu < 0) + \beta\sigma_{t-1}^2
\end{aligned} \tag{7}
$$

where I is a (0,1) indicator function. The problem with the threshold ARCH model above is that coefficient α may take negative values in practice for equity returns. In the new model by allowing both ARCH and GARCH parameters to change when $y_{t-1} - \mu < 0$ the problem of negative bias is resolved.

Generalized Threshold GARCH or GTARCH(1,1,1,1)

$$
\begin{aligned}
y_t &= \mu + \epsilon_t \\
\sigma_t^2 &= \omega + \alpha\epsilon_{t-1}^2 + \gamma\epsilon_{t-1}^2 I(y_{t-1} - \mu < 0) + \beta\sigma_{t-1}^2 + \delta\sigma_{t-1}^2 I(y_{t-1} - \mu < 0)
\end{aligned} \tag{8}
$$

The Stationarity of GTARCH Model

The new GTARCH model shows higher persistence for negative returns (compared to positive returns) in terms of both γ (yesterday's shock) and δ (previous volatility forecast).

The weak stationarity condition in the GARCH model for the existence of the long run unconditional variance σ^2 is given by condition:

$$\alpha + \beta < 1, \quad \sigma^2 = \frac{\omega}{1 - \alpha - \beta}$$

Similarly for the GTARCH model we can define $\theta = E(I(y_t < \mu))$.
The weak stationarity condition and the unconditional variance are given by

$$\alpha + \beta + \gamma\theta + \delta\theta < 1, \quad \sigma^2 = \frac{\omega}{1 - \alpha - \beta - \gamma\theta - \delta\theta}$$

MCMC Estimation

Let the prior probability for the GTARCH volatility model be given by

$$
\begin{aligned}
\pi(\mu, \alpha, \gamma, \beta, \delta) \quad &\propto \quad \mathbf{N}(\mu_0, \Sigma_\mu)\, \mathbf{N}(\alpha_0, \Sigma_\alpha)\, \mathbf{N}(\gamma_0, \Sigma_\gamma) \\
&\times \quad \mathbf{N}(\beta_0, \Sigma_\beta)\, \mathbf{N}(\delta_0, \Sigma_\delta)
\end{aligned}
\qquad (9)
$$

where $\mu, \alpha, \gamma, \beta$ and δ are the GTARCH parameters and have proper normal priors with large variances.

Consider the Dynamic Conditional Correlations (DCC) model with GTARCH volatility. The posterior pdf of DCC model is

$$
p(\eta_1, \eta_2, \psi | data) \propto \pi(\eta_1, \eta_2, \psi) \times L(data | \eta_1, \eta_2, \psi) \qquad (10)
$$

$$
\eta_i = \mu_i, \alpha_i, \gamma_i, \beta_i, \delta_i
$$

$$
\psi = \omega_{ij}, \alpha, \beta
$$

Let n=2 (2 firms, and a market).
The DCC log likelihood is given by

$$logL = log(L_v(\eta_1, \eta_2) + log(L_c(\eta_1, \eta_2, \psi) \tag{11}$$

$$log(L_v) = -0.5 \sum (nlog(2\pi) + log(\sigma_{i,t}^2) + \frac{r_{i,t}^2}{\sigma_{i,t}^2}) \tag{12}$$

$$\begin{aligned} log(L_c) &= -0.5 \sum \left(log(1-\rho_{12,t}^2) + \frac{z_{1,t}^2 + z_{2,t}^2 - 2\rho_{12,t} z_{1,t}^2 z_{2,t}^2}{1-\rho_{12,t}^2} \right) && (13) \\ \rho_{12,t} &= \frac{q_{12,t}}{\sqrt{q_{11,t} q_{22,t}}} && (14) \\ q_{ij,t} &= \omega_{ij}(1-\alpha-\beta) + \alpha z_{i,t} z_{j,t} + \beta q_{ij,t-1} && (15) \\ & && (16) \end{aligned}$$

where $r_{i,t}$ and $r_{m,t}$ are daily log returns of firm i and the market correspondingly. The standardized returns: $z_{i,t} = \frac{r_{i,t}}{\sqrt{h_{it}}}$

MCMC steps and Data used in the study

Step 1: I estimate parameters in blocks for each asset GTARCH model using random walk draws.

Step 2: using fitted volatilities from step 1 find standardized returns z_{it} and estimate dynamic correlation between two assets. We estimate parameters in blocks using random walk draw: (i) ARCH parameters: α and ω_{12} as part of ARCH, (ii) GARCH parameters β, (iii) Constant terms $\omega_{ii} = 1 - \alpha - \beta$ for i=1,2.

The data are from CRSP for returns and market capitalization for the period 2001/01/02-2012/12/31. The book value of debt is from COMPUSTAT.

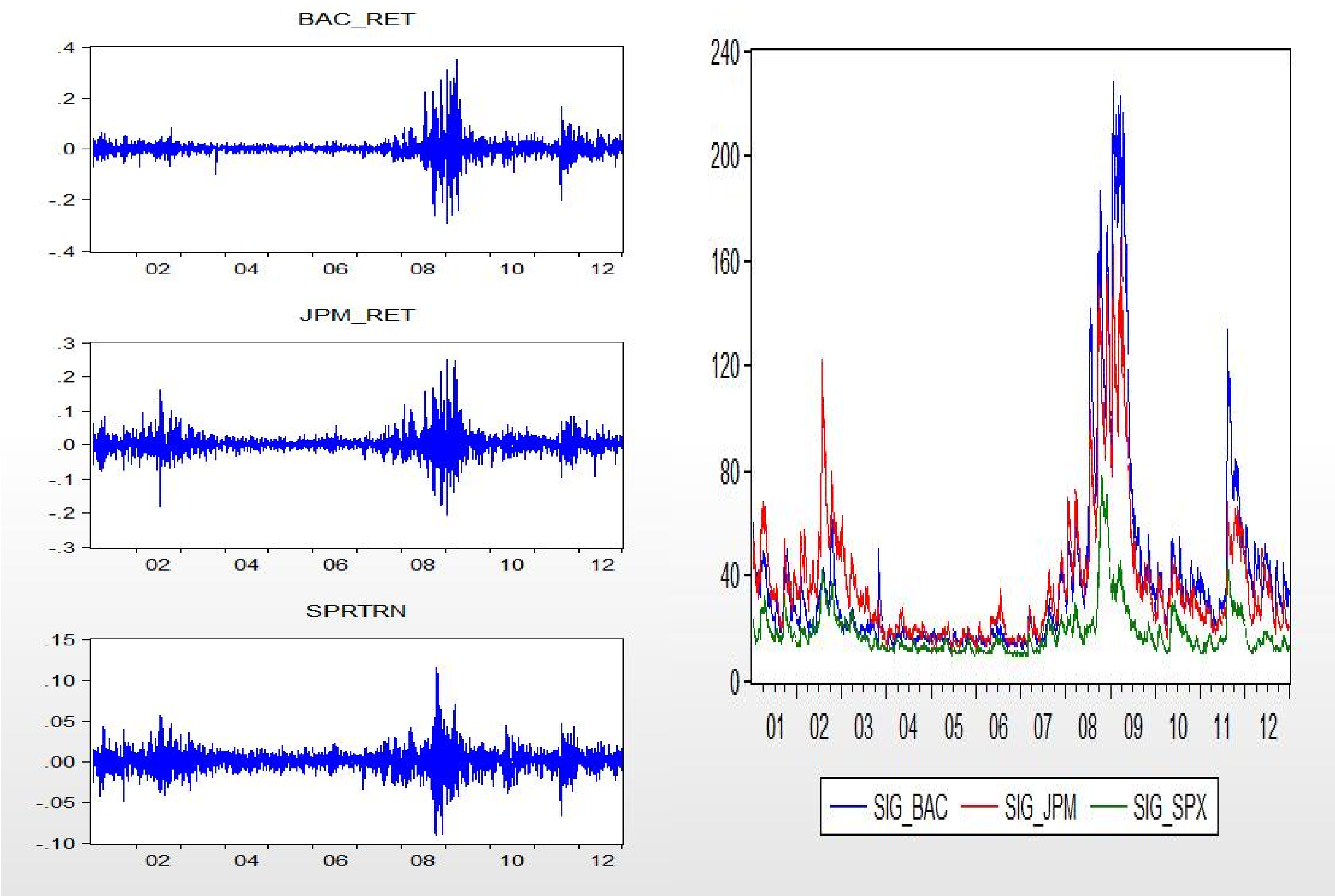
Data Analysis of MES and SRISK for 10 systemically important US institutions

The following 10 systemically important financial firms were ranked the highest on VLAB website as of June 7, 2013 (see Table 4).

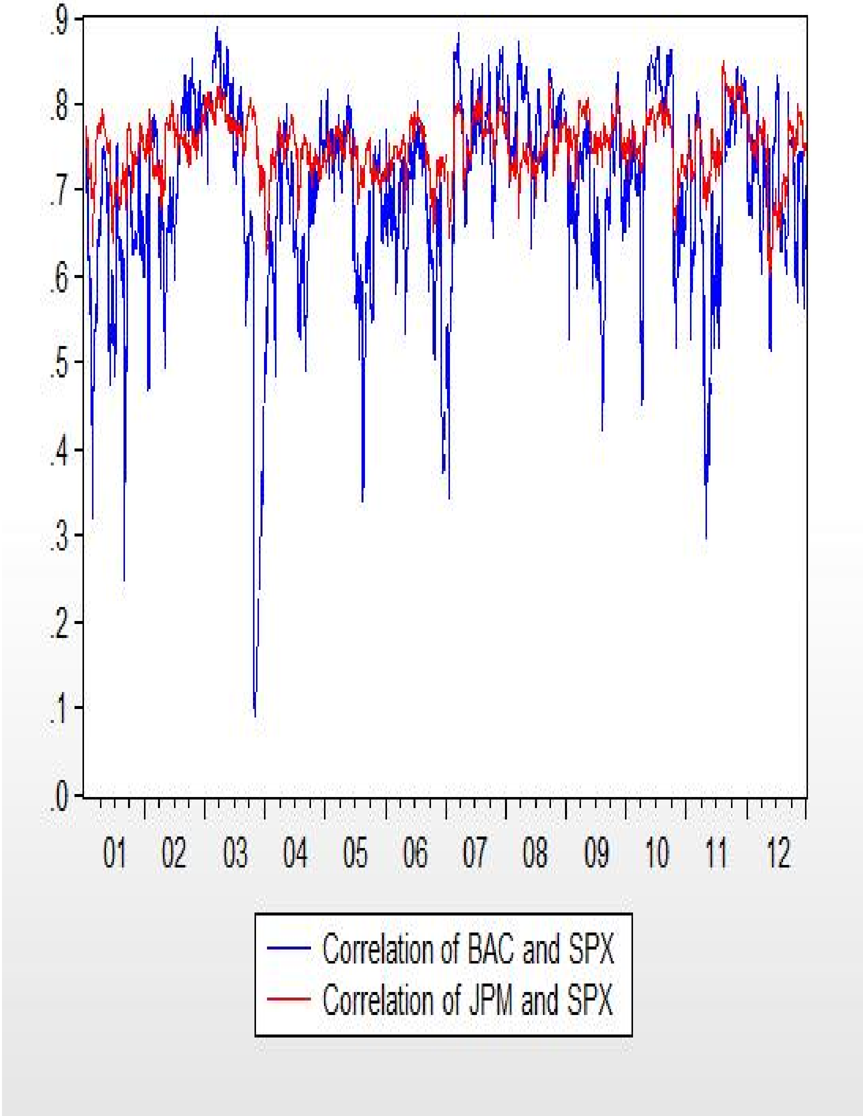
US Top 10 SRISK	SRISK%	LRMES	LVG
Bank Of America	16.5	51.46	14.16
JP Morgan Chase	16.1	54.12	11.58
Citigroup	13.4	57.24	11.66
MetLife	8.7	66.29	17.04
Prudential Financial	8.1	63.04	22.26
Morgan Stanley	8.0	67.76	15.40
Goldman Sachs	6.4	49.50	12.42
Hartford Financial Services	3.1	57.51	20.77
Capital One Financial	3.0	86.21	8.27
Lincoln National Corp	2.6	69.13	22.88

Source: http://vlab.stern.nyu.edu on June 7, 2013

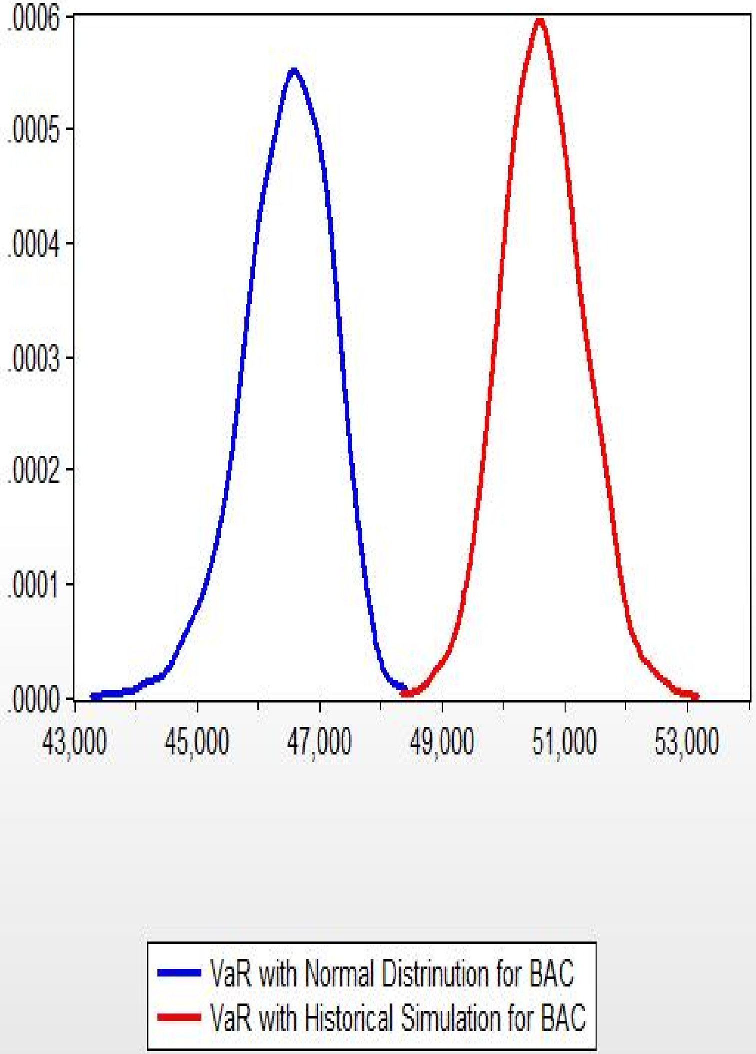
Returns and Estimated TGARCH volatility: BAC, JPM, SPX

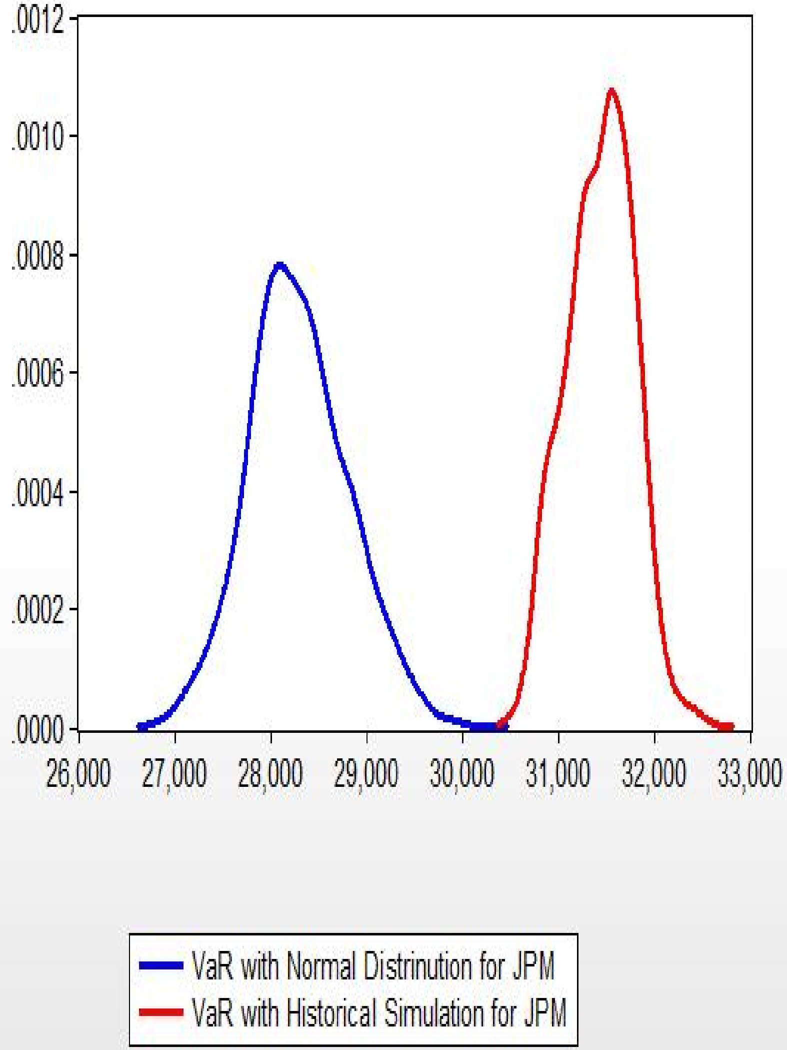


Dynamic correlation with the market: BAC, JPM



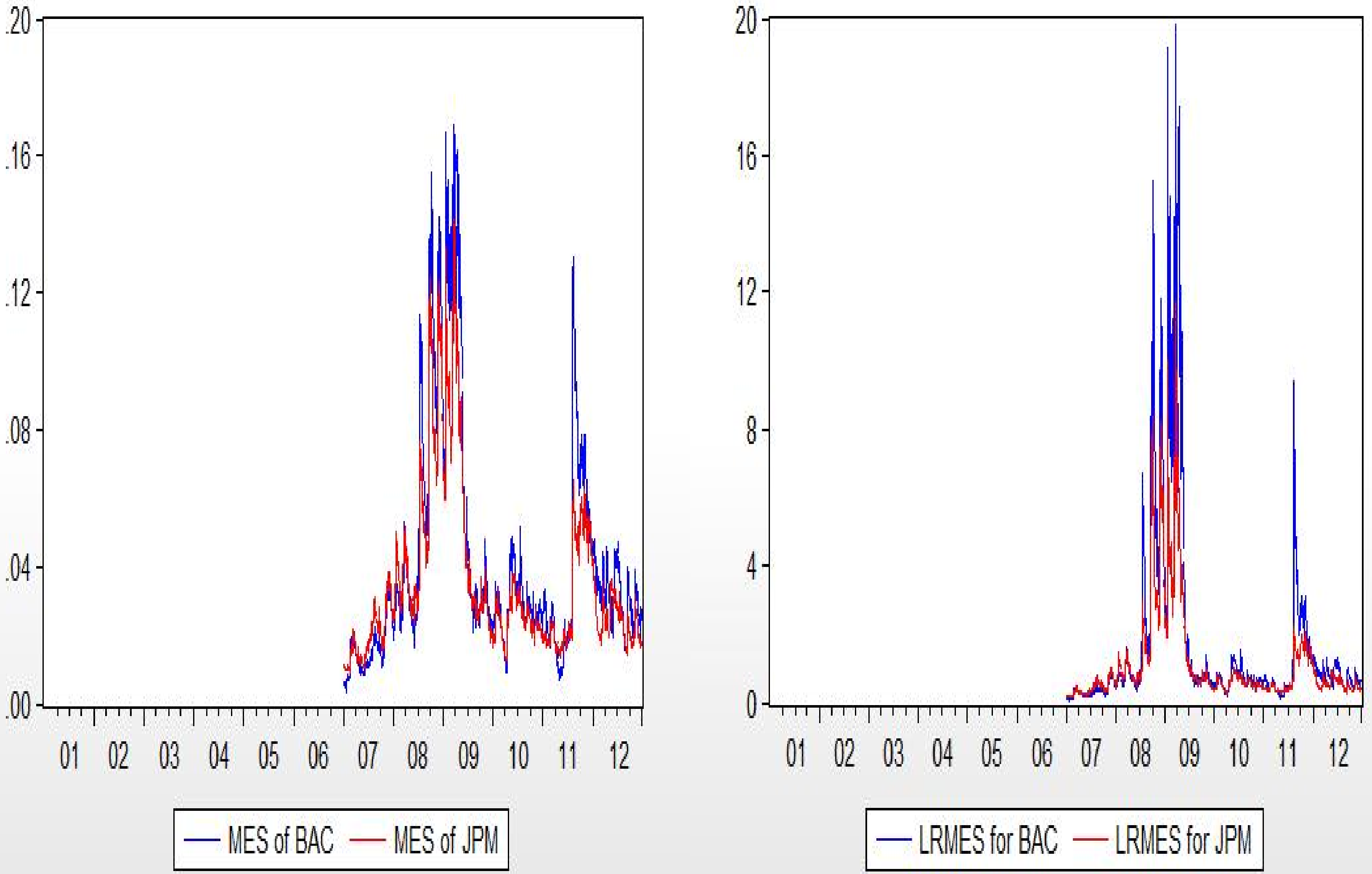
Distribution of 1% quantile (VaR) for the $1 million portfolio if (a) the residuals are normal and (b) corrected for fat tails: BAC, JPM



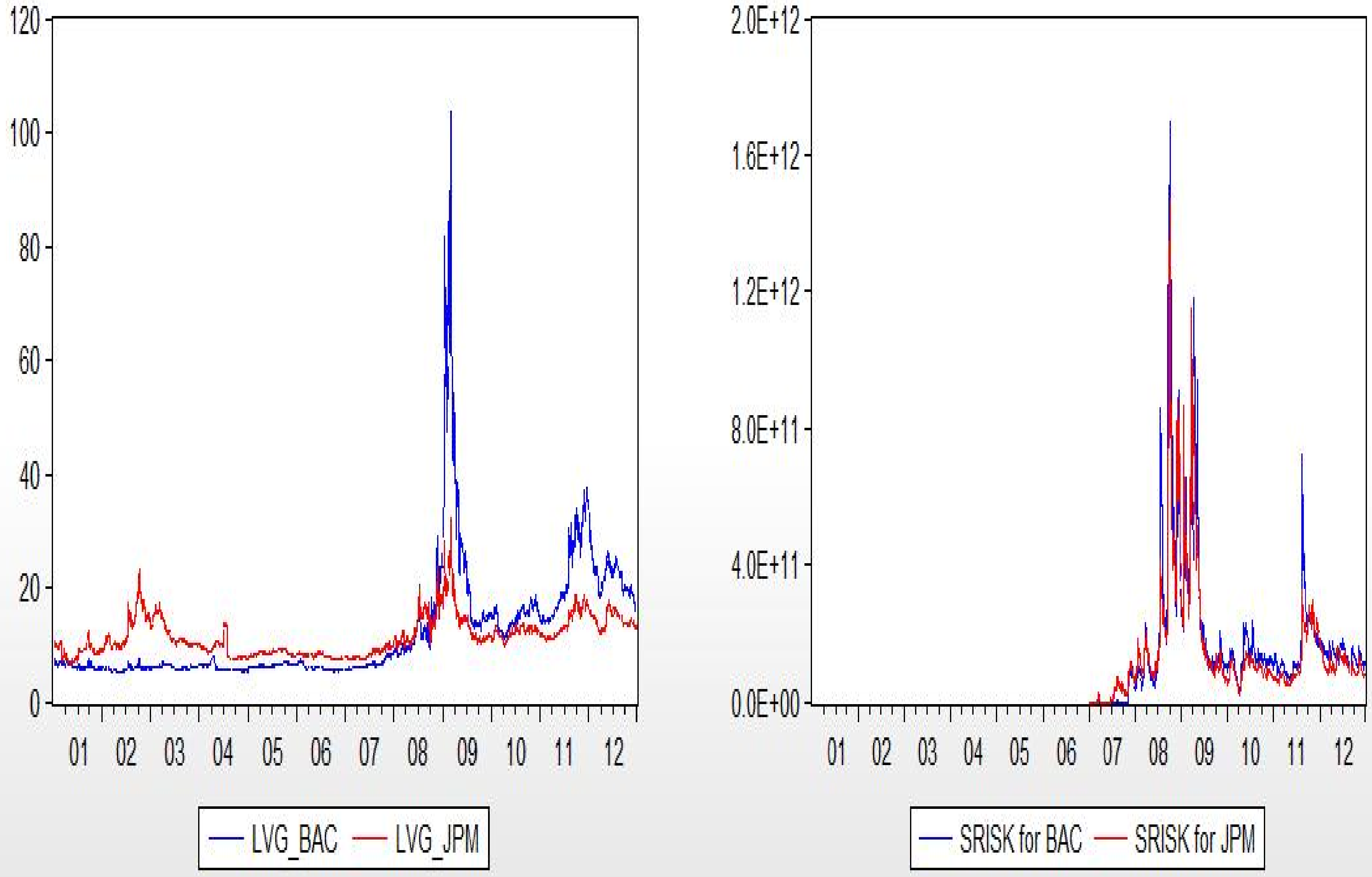


Marginal Expected Shortfall (MES) and Long Run

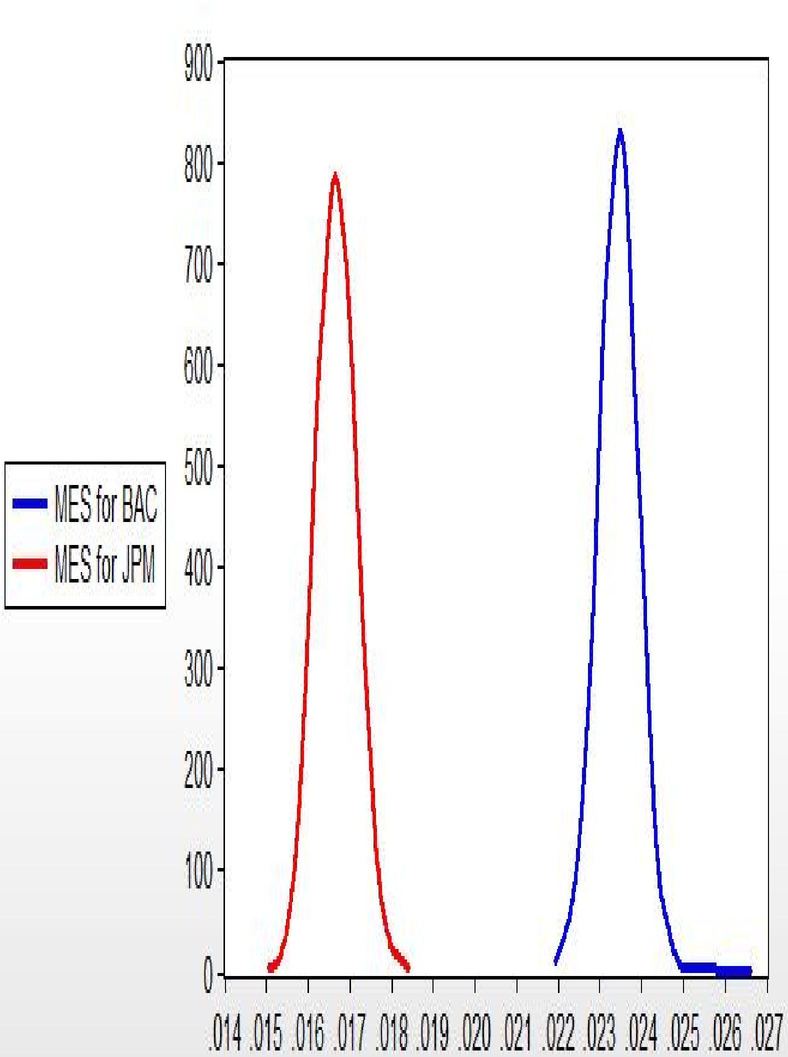
MES: BAC, JPM

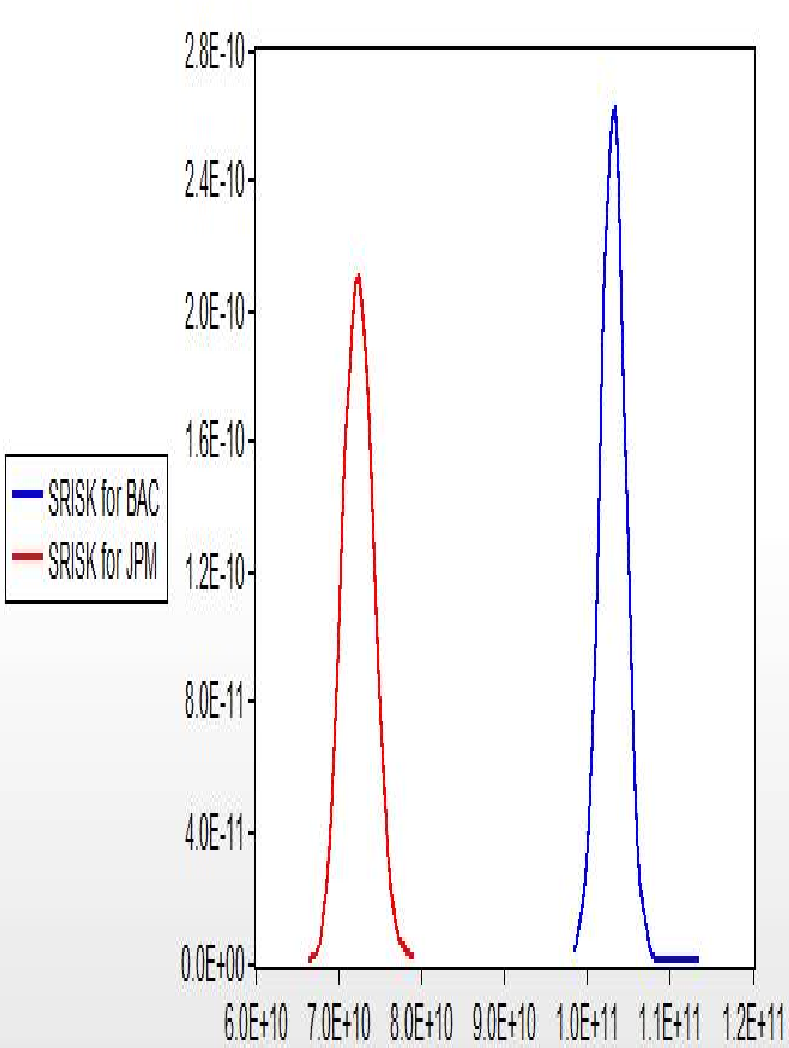


Leverage and SRISK: BAC and JPM



Posterior PDFs of Marginal Expected Shortfall and SRISK: BAC, JPM





Conclusion

Using a new asymmetric GTARCH model and capturing uncertainty around the measures I found that MES, LRMES and SRISK are statistically different for major financial firms in the periods pf low volatility, but not in periods of high volatility.

1. Introduced Bayesian analysis of the systemic risk measures, derived the full posterior distributions and showed how to distinguish risks of different institutions.
2. Introduced and estimated a new asymmetric GTARCH model that corrects the caveat and generalizes popular asymmetric volatility GJR-GARCH model.
3. Future work: consider different distributional assumptions for the error term and compare the market based measures of systemic risks used in this paper to the results of macroprudential stress tests.