Dynamic Analysis of 'Too Big to Fail' Risks

This draft: December, 2013

Abstract

In this paper I introduce a generalized threshold conditional volatility model (GTARCH) which captures asymmetry in both ARCH and GARCH terms. The GTARCH is then combined with DCC (dynamic conditional correlations model) in order to measure MES (marginal expected shortfall) and SRISK (the expected capital shortage of a firm). Markov Chain Monte Carlo (MCMC) algorithms are used for estimation of both models and the distributions of systemic risks are derived from MCMC draws. The GTARCH model is shown to overperform the standard GJR-GARCH asymmetric volatility model for equity returns and log-differences of Credit Default Swaps (CDS) for banks' secured bonds. The CDS spreads and their volatility are used as additional measures of systemic risks of financial institutions. Overall, I found that the distributions of systemic risks are statistically different for major financial institutions in the recent period of low volatility but are very close during the financial crisis. Thus, at the time when volatility is high it is hard to rank banks based on MES and SRISK that are reported by the Volatility Institute (VLAB).

1 Introduction

Following recent studies of systemic risks by Acharya et. al (2010) and Brownlees and Engle (2012) among others I introduce Bayesian estimation of MES (marginal expected shortfall) and SRISK (the expected capital shortage of a firm conditional on a substantial market decline). The rankings for MES and SRISK are used to analyze the systemic risks of financial institutions and are daily reported by the Volatility Institute[1]. However, this measures are reported without uncertainty around the estimates and thus one cannot distinguish if the difference in rankings of large financial institutions is statistically significant. Recent surveys of systemic risk analytics by

[1] See http://vlab.stern.nyu.edu

Bisias et al. (2012) and Brunnermeier and Oehmke (2012) among others also do not show how to measure and incorporate uncertainty for systemic risk measures. To fill this gap the present paper will show how to estimate MES and SRISK using Bayesian Markov Chain Monte Carlo (MCMC) algorithms.

In this paper I also introduce a generalized threshold conditional volatility model (GTARCH) and compare it to traditional asymmetric models of volatility. Since introduction of the generalized autoregressive conditional heteroscedasticity (GARCH) model there have been many extensions of GARCH models that resulted in better statistical fit and forecasts. For example, GJR-GARCH (Glosten, Jagannathan, & Runkle (1993)) is one of the well-known extensions of GARCH models with an asymmetric term which captures the effect of negative shocks in equity prices on volatility commonly referred to as a "leverage" effect. The widely used GJR-GARCH model has a problem that one of the estimated coefficients of the volatility model (alpha) takes a meaningless negative value for equity indices. The typical solution to this problem is setting the coefficient of alpha to zero in the constrained Maximum Likelihood optimization.

In the GTARCH model both coefficients, ARCH (α) and GARCH (β), are allowed to change to reflect the asymmetry of volatility due to negative shocks. As a subset of this model GJR-GARCH model allows for asymmetry only in ARCH. Alternatively, the GTARCH model allows for asymmetry only in GARCH or no asymmetry. Theoretically and using Monte Carlo experiments it can be shown that additional asymmetric GARCH term shifts the value of α upward compared to the GJR-GARCH model. The asymmetric GARCH term compared to the asymmetric ARCH term makes α positive. The suggested more flexible GTARCH model also shows more persistent dynamics for GARCH parameters for the negative news and lower persistence for the positive news. Our results for equity returns show that compared to GJR-GARCH and GARCH our model predicts higher level of volatility in high volatility periods and lower levels of volatility in low volatility periods.

The GTARCH is then combined with DCC (dynamic conditional correlations model) in order to measure MES and SRISK. Brownlees and Engle (2012) use the Maximum Likelihood estimation of GJR-GARCH volatility models for market and firm returns as well as DCC model for the tail dependence. MES can be derived as a function of volatility, correlation and tail expectations of a firm and market return innovations. When measuring tail expectation Brownlees and Engle (2012) use nonparametric kernel estimation without incorporating uncertainty. In this paper Markov Chain Monte Carlo (MCMC) algorithms are used for estimation of both models and the distributions of systemic risks are derived from MCMC draws. The advantage of Markov Chain Monte Carlo algorithms is the natural ability to generate the posterior predictive densities for variables of interest, such as volatility, correlation, value at risk, expected shortfall, etc. The algorithms for estimating the GTARCH model and its combination with the the Dynamic Conditional Correlations DCC-GTARCH are based on the extension of previously published algorithms of Goldman and Tsurumi (2005). I use Metropolis-Hastings steps with random walk draws.

The Credit Default Swaps (CDS) data are widely used to access the default risks of financial institutions and sovereign bonds. The literature analysing the risks implied by CDS is growing. For example, Hull et. al (2004) among others studied the relation between CDS spreads, bond yield spreads and credit rating announcements. Carr and Wu (2011) show the relation between CDS spreads and out-of-the-money American put options. In this paper I estimate the GTARCH model for the log-differences of CDS spreads and find that the asymmetry resulting from higher spread is better explained by the GTARCH than the GJR-GARCH model. The CDS premiums change dramatically over time and may exhibit nonstationary behaviour. It can be argued that the systemic risks of financial institutions can be related to the level and volatility of CDS premiums. Some work in this direction was recently done by Oh and Patton (2013) and the Volatility Institute where the CDS GARCH results are now available.

The remainder of the paper is organized as follows. Section 2 presents the measurements of the systemic risks and section 3 presents the GTARCH model. Section 4 presents summary statistics of the data and MLE results of GTARCH model for S&P500 returns and log-differences in CDS premiums for Bank of America (BAC) and JP Morgan Chase (JPM). Section 5 presents the MCMC algorithms. Section 6 estimates models using MCMC for the MES and SRISK of BAC and JPM overtime and distribution of these measures in periods of high and low volatility for different volatility models. Section 7 concludes.

2 Measurement of Systemic Risk

Let r_t and $r_{m,t}$ be the daily log returns of a firm and the market correspondingly. Following Brownlees and Engle (2012) we consider the following model for the returns:

$$\begin{aligned} r_{mt} &= \sigma_{mt}\epsilon_{mt} \\ r_t &= \sigma_t \rho_t \epsilon_{mt} + \sigma_t \sqrt{1-\rho_t^2}\epsilon_t \end{aligned} \tag{1}$$

where $\epsilon_{mt}, \epsilon_t \sim F$ are independent and identically distributed variables with zero means and unit variances, σ_t and σ_{mt} are conditional standard deviations of the firm return and the market return correspondingly, and ρ_t is conditional correlation between the firm and the market. This model is also called the dynamic conditional beta model with $\beta_t = \rho_t \dfrac{\sigma_t}{\sigma_{mt}}$ and tail dependence on correlation of firm returns and the market

$$r_t \;=\; \beta_t r_{mt} + \sigma_t \sqrt{1-\rho_t^2}\epsilon_t \tag{2}$$

The conditional variances and correlation are modelled using the GJR-GARCH DCC model in Brownlees and Engle (2012). In the next section I introduce the generalized threshold GARCH volatility model and show that it outperforms GJR-GARCH for equities.

In this paper I only consider the market based measures of systemic risks. Other macroprudential and microprudential tests are beyond the scope of this paper but are described in Bisias, Flood, Lo and Valavanis (2012) and Acharya, Engle and Pierret (2013) among others.

The first considered systemic risk measure is the daily marginal expected shortfall (MES) which is the conditional expectation of a daily return of a financial institution given that the market return falls below threshold level C. In practice, in VLAB it is assumed that market falls by more than 2%, i.e. the threshold $C = -2\%$.

$$\begin{aligned} MES_{t-1} &= E_{t-1}(r_t|r_{mt} < C) \qquad (3)\\ &= \sigma_t \rho_t E_{t-1}(\epsilon_{mt}|\epsilon_{mt} \le C/\sigma_{mt}) + \sigma_t\sqrt{1-\rho_t^2}E_{t-1}(\epsilon_t|\epsilon_{mt} \le C/\sigma_{mt}) \end{aligned}$$

The computation of the expected shortfall following Scaillet (2005) using nonparametric estimates given by:

$$E_{t-1}(e_{mt}|e_{mt} \le \alpha) = \frac{\sum_{i=1}^{t-1} e_{mi}\Phi_h(\frac{\alpha - e_{mi}}{h})}{\sum_{i=1}^{t-1}\Phi_h(\frac{\alpha - e_{mi}}{h})} \tag{4}$$

where $\alpha = C/\sigma_{mt}$, $\Phi_h(t) = \int_{-\infty}^{t/h}\phi(u)du$, $\phi(u)$ is a standard normal probability distribution function used as kernel, and $h = T^{-1/5}$ is the bandwidth parameter.

The second measure is the long run marginal expected shortfall based on the expectation of the cumulative six month firm return conditioned on the event that the market falls by more than 40% in six months. It is shown by Acharya, Engle and Richardson (2012) that the LRMES can be approximated by

$$LRMES_t \approx 1 - e^{-18*MES_t} \tag{5}$$

Finally, the capital shortfall of the firm based on the potential capital loss in six months is defined as

$$SRISK_t = max\{0; kD_t - (1-k)(1-LRMES_t)E_t\} \tag{6}$$

where D_t is the book value of Debt at time t, E_t is the market value of equity at time t and $k \approx 8\%$ is the prudential capital ratio of the US banks. It is assumed that the capital loss happens only due to the loss in the market capitalization $LRMES * E_t$

3 Generalized Threshold GARCH model

GJR-GARCH (Glosten, Jagannathan, & Runkle (1993)) is one of the well-known asymmetric volatility models which captures the effect of negative shocks in equity prices on volatility commonly referred to as a "leverage" effect. The model captures risk-aversion of investors with volatility increasing more as a result of a negative news compared to the positive news.[2]

Consider the GJR-GARCH volatility model for returns r_t with mean μ given in equation (7) below.

GJR-GARCH(1,1,1)

$$\begin{aligned} r_t &= \mu + \epsilon_t \\ \sigma_t^2 &= \omega + \alpha\epsilon_{t-1}^2 + \gamma\epsilon_{t-1}^2 I(r_{t-1} - \mu < 0) + \beta\sigma_{t-1}^2 \end{aligned} \tag{7}$$

where I is a (0,1) indicator function, σ_t is conditional volatility.

The Generalized Threshold GARCH (GTARCH) model that I introduce in equation (8) is an extension of the model above allowing GARCH term to change for a negative news ($\epsilon_{t-1} < 0$).

GTARCH(1,1,1,1)

$$\begin{aligned} r_t &= \mu + \epsilon_t \\ \sigma_t^2 &= \omega + \alpha\epsilon_{t-1}^2 + \gamma\epsilon_{t-1}^2 I(r_{t-1} - \mu < 0) + \beta\sigma_{t-1}^2 + \delta\sigma_{t-1}^2 I(r_{t-1} - \mu < 0) \end{aligned} \tag{8}$$

The Stationarity of GTARCH Model

The weak stationarity condition in the GARCH model for the existence of the long run unconditional variance σ^2 is given by condition:

$$\alpha + \beta < 1, \qquad \sigma^2 = \frac{\omega}{1 - \alpha - \beta}$$

[2] EGARCH is an alternative model but it is in logs of variance rather than typical GARCH variance.

Similarly for the GTARCH model we can define $\theta = E(I(r_t < \mu))$ which is percentage of observations with $r_t < \mu$. Then the weak stationarity condition and the unconditional variance are given by

$$\alpha + \beta + \gamma\theta + \delta\theta < 1, \quad \sigma^2 = \frac{\omega}{1 - \alpha - \beta - \gamma\theta - \delta\theta}$$

4 Data description and MLE results

In this section I consider the equity returns daily data for BAC, JPM and S&P 500 index for the period 1/04/2001-12/31/2012 from CRSP database. I also consider the CDS spreads on the 5 year secured bonds of BAC and JPM for the period 9/06/2001-10/08/2013 from Bloomberg. All these data will be used for the analysis of systemic risks in Section 6.

The summary statistics of the data are given in Table 1. All the series have fat tails with the kurtosis over 10 and some skewness. Even though the CDS spreads typically have significant positive skewness the log-differences of CDS spreads for BAC and JPM do not show considerable skewness.

There may be some autocorrelation present in the model although AR(1) coefficients are not large.

Table 1: Summary statistics for daily equity returns and log-differences of daily CDS spreads

	BAC	JPM	S&P 500	CDS BAC	CDS JPM
mean	0.045	0.047	0.011	0.0447	0.0287
std	3.406	2.841	1.342	5.0053	4.2022
Skew	0.904	0.829	0.017	-0.2013	-0.1753
Kurt	26.08	15.931	11.143	14.3665	16.5461
AR(1)	-0.011	-0.089	-0.091	-0.021	0.052

Notes: Equity returns and log differences in CDS spreads for BAC (Bank Of America) and JPM (JP Morgan Chase). All measured in basis points. Equity prices data are for the period 1/04/2001-12/31/2012 from CRSP database. CDS data are for the period 9/06/2001-10/08/2013 from Bloomberg.

We consider the GTARCH model for the returns and log-differences of CDS spreads. Unlike for the equity returns the bad news in CDS market is when the spreads increase. Thus, we change the sign of the error in the dummy indicator function to $I(r_{t-1} - \mu > 0)$ for the CDS data.

Tables 2-4 shows results of estimation using maximum likelihood for the S&P500 returns and log-differences of CDS spreads for BAC and JPM. Using information criteria we can see that the GTARCH model or a simpler version of it with $\gamma = 0$ is preferable in most cases. If the γ asymmetry is not used in the model all coefficients are positive.

The new GTARCH model shows higher persistence for negative returns (compared to positive returns) in terms of both γ (yesterday's shock) and δ (previous volatility forecast). Compared to the GJR-GARCH model the effect of γ is smaller because of the presence of δ in the model.

All models show high persistence measured by $\alpha + \beta + .5(\gamma + \delta)$.

Table 2: MLE for different specifications of the GTARCH model for SPX returns (01/04/2001- 12/31/2012)

	GTARCH	GJR-GARCH ($\delta = 0$)	GTARCH ($\gamma = 0$)	GARCH ($\gamma = 0, \delta = 0$)
μ	0.006 (0.0004)	0.003 (0.016)	0.012 (0.016)	0.048 (0.016)
ω	0.017 (0.002)	0.013 (0.002)	0.020 (0.002)	0.016 (0.002)
α	-0.015 (0.007)	-0.026 (0.006)	0.068 (0.007)	0.087 (0.008)
γ	0.130 (0.011)	0.155 (0.011)		
β	0.885 (0.012)	0.938 (0.007)	0.816 (0.012)	0.903 (0.008)
δ	0.110 (0.018)		0.215 (0.018)	
$\alpha + \beta + .5(\gamma + \delta)$	0.990	0.989	0.992	0.990
AIC	2.884*	2.890	2.912	2.939
SIC	2.896*	2.900	2.922	2.947

Table 3: MLE for different specifications of the GTARCH model for log-differences of CDS spread for JPM (09/11/2001- 08/12/2013)

	GTARCH	GJR-GARCH ($\delta = 0$)	GTARCH ($\gamma = 0$)	GARCH ($\gamma = 0, \delta = 0$)
μ	-0.067 (0.0615)	-0.093 (0.061)	-0.131 (0.019)	-0.113 (0.057)
ω	0.332 (0.028)	0.234 (0.019)	0.244 (0.023)	0.224 (0.019)
α	0.113 (0.008)	0.094 (0.006)	0.109 (0.004)	0.113 (0.004)
γ	0.023 (0.010)	0.030 (0.008)		
β	0.851 (0.007)	0.892 (0.003)	0.868 (0.006)	0.889 (0.003)
δ	0.039 (0.011)		0.046 (0.010)	
$\alpha + \beta + .5(\gamma + \delta)$	0.995	1.000	0.999	1.003
AIC	5.4517	5.4514	5.4508*	5.4518
SIC	5.4635	5.4612	5.4605	5.4596*

Table 4: MLE for different specifications of the GTARCH model for log-differences of CDS spread for BAC (09/11/2001- 08/12/2013)

	GTARCH	GJR-GARCH ($\delta = 0$)	GTARCH ($\gamma = 0$)	GARCH ($\gamma = 0, \delta = 0$)
μ	-0.095 (0.0637)	-0.044 (0.066)	-0.097 (0.049)	-0.097 (0.055)
ω	0.352 (0.027)	0.334 (0.027)	0.348 (0.028)	0.327 (0.027)
α	0.121 (0.009)	0.089 (0.007)	0.117 (0.005)	0.119 (0.005)
γ	0.012 (0.013)	0.046 (0.010)		
β	0.836 (0.008)	0.884 (0.004)	0.840 (0.007)	0.879 (0.004)
δ	0.065 (0.011)		0.071 (0.009)	
$\alpha + \beta + .5(\gamma + \delta)$	0.995	0.996	0.993	0.998
AIC	5.7394	5.7410	5.7389*	5.7418
SIC	5.7511	5.7508	5.7487*	5.7496

5 Markov Chain Monte Carlo Algorithms

Markov Chain Monte Carlo (MCMC) algorithms allow to estimate posterior distributions of parameters by simulation and are especially useful when the dimension of parameters is high, since the problems of multiple maxima or of initial starting values are avoided. A simple intuitive explanation of the Metropolis-Hastings algorithm is given in Chib and Greenberg (1995).

MCMC algorithms were developed by Chib and Greenberg (1994) for the ARMA model and by Nakatsuma (2000) and Goldman and Tsurumi (2005) for the ARMA-GARCH model. Chib and Greenberg (1994) (as well as Nakatsuma (2000)) use the constrained nonlinear maximization algorithm in the MA block. Alternatively one can use a Metropolis-Hastings algorithm with a random walk Markov Chain as was done e.g. in Goldman and Tsurumi (2005). The random walk draws speed up the computational time of the MCMC algorithms without losing much of the acceptance rate of the Metropolis-Hastings algorithm. In this paper we propose the algorithms for a GTARCH model which is an extension of the algorithms developed in Goldman and Tsurumi (2005).

Let the prior probability for the GTARCH volatility model be given by

$$\begin{aligned} \pi(\mu, \alpha, \gamma, \beta, \delta) &\propto \mathrm{N}(\mu_0, \Sigma_\mu)\, \mathrm{N}(\alpha_0, \Sigma_\alpha)\, \mathrm{N}(\gamma_0, \Sigma_\gamma) \\ &\times \mathrm{N}(\beta_0, \Sigma_\beta)\, \mathrm{N}(\delta_0, \Sigma_\delta) \end{aligned} \tag{9}$$

where μ, α, γ, β and δ are the GTARCH parameters and have proper normal priors with large variances.

Consider the Dynamic Conditional Correlations (DCC) model with GTARCH volatility. The posterior pdf of DCC model is

$$p(\eta_1, \eta_2, \psi | data) \propto \pi(\eta_1, \eta_2, \psi) \times L(data|\eta_1, \eta_2, \psi) \tag{10}$$

$$\eta_i = \mu_i, \alpha_i, \gamma_i, \beta_i, \delta_i$$

$$\psi = \omega_{ij}, \alpha, \beta$$

Let n=2 (2 firms, and a market).

The DCC log likelihood is given by

$$logL = log(L_v(\eta_1, \eta_2) + log(L_c(\eta_1, \eta_2, \psi) \tag{11}$$

$$log(L_v) = -0.5 \sum (nlog(2\pi) + log(\sigma_{i,t}^2) + \frac{r_{i,t}^2}{\sigma_{i,t}^2}) \tag{12}$$

$$\begin{aligned} log(L_c) &= -0.5 \sum \left(log(1 - \rho_{12,t}^2) + \frac{z_{1,t}^2 + z_{2,t}^2 - 2\rho_{12,t} z_{1,t}^2 z_{2,t}^2}{1 - \rho_{12,t}^2} \right) & (13) \\ \rho_{12,t} &= \frac{q_{12,t}}{\sqrt{q_{11,t} q_{22,t}}} & (14) \\ q_{ij,t} &= \omega_{ij}(1 - \alpha - \beta) + \alpha z_{i,t} z_{j,t} + \beta q_{ij,t-1} & (15) \\ & & (16) \end{aligned}$$

where $r_{i,t}$ and $r_{m,t}$ are daily log returns of firm i and the market correspondingly. The standardized returns: $z_{i,t} = \frac{r_{i,t}}{\sqrt{h_{it}}}$

Step 1: I estimate parameters in blocks for each asset GTARCH model using random walk draws.

Step 2: using fitted volatilities from step 1 find standardized returns z_{it} and estimate dynamic correlation between two assets. We estimate parameters in blocks using random walk draw: (i) ARCH parameters: α and ω_{12} as part of ARCH, (ii) GARCH parameters β, (iii) Constant terms $\omega_{ii} = 1 - \alpha - \beta$ for i=1,2.

Each step is a separate MCMC chain and careful tests of convergence are applied.[3]

[3] I use the graphs of draws, fluctuation test (see Goldman and Tsurumi (2005)) and the acceptance rates to judge convergence. The results are available from author on request.

6 Data Analysis of MES and SRISK for systemically important US institutions

We consider Bank of America and JP Morgan Chase ranked in the top three highest systemically important financial firms on VLAB website as of December 31,2012-June 7, 2013 (Tables 5-6).

Table 5: VLAB Systemic Risks for US top 10 institutions on December 31, 2012

US Top 10 SRISK	SRISK%	RNK	SRISK ($ m)	MES	Beta	Cor	Vol	Lvg	MV
Bank of America	19.14	1	101,084	4.3	1.54	0.67	28.8	16.4	125,133.50
Citigroup	15.87	2	83,808	3.62	1.37	0.69	24.9	16.02	116,010.50
JP Morgan Chase	14.36	3	75,859	2.74	1.11	0.74	18.8	13.69	167,144.20
MetLife	8.92	4	47,121	4.24	1.59	0.7	28.6	22.75	35,939.00
Goldman Sachs	7.72	5	40,755	3.71	1.41	0.72	24.6	15.14	61,817.2
Prudential Financial	7.2	6	38,036	3.33	1.38	0.75	23.2	26.44	24,851.80
Morgan Stanley	7.12	7	37,589	3.64	1.42	0.69	25.9	19.42	37,749.00
Hartford Financial Serv	3.4	8	17,950	3.39	1.42	0.68	26.3	30.17	9,790.80
American Intern Group	2.41	9	12,709	4.05	1.41	0.6	29.4	9.6	52,113.50
Lincoln National Corp	2.38	10	12,584	3.59	1.35	0.69	24.7	29.11	7,122.90

Source: http://vlab.stern.nyu.edu on December 31, 2012

Table 6: VLAB Systemic Risks for US top 10 institutions on June 7, 2013

US Top 10 SRISK	SRISK%	LRMES	LVG
Bank Of America	16.5	51.46	14.16
JP Morgan Chase	16.1	54.12	11.58
Citigroup	13.4	57.24	11.66
MetLife	8.7	66.29	17.04
Prudential Financial	8.1	63.04	22.26
Morgan Stanley	8.0	67.76	15.40
Goldman Sachs	6.4	49.50	12.42
Hartford Financial Services	3.1	57.51	20.77
Capital One Financial	3.0	86.21	8.27
Lincoln National Corp	2.6	69.13	22.88

Source: http://vlab.stern.nyu.edu on June 7, 2013

For the systemic risk modeling as in Brownlees and Engle (2012) I use market data on stock prices, market capitalization and book value of debt for large financial institutions. The data are from CRSP for returns and market capitalization for the period 2001/01/02-2012/12/31. The book value of debt is from COMPUSTAT.

Figure 1 shows the returns data for BAC, JPM and SPX. The dynamic GTARCH volatility estimated at posterior means of parameters is plotted in Figure 2. While before the financial crisis JPM had higher level of volatility, during the crisis and after the crisis BAC volatility level exceeded JPM. Not surprisingly the SPX has lower equity volatility then both banks. The dynamic correlation of firms with the market also estimated at posterior means of parameters is given in Figure 3. For comparison I also present 100-day rolling correlations in Figure 4. Both graphs show changing patterns of correlation over time with less variability for the DCC-GTARCH model.

After the equity volatility models were estimated for each bank I found the distributions of 1% Value at Risk (VaR) and showed them in Figure 5 for a $1 million portfolio using (a) Normal distribution for the error term and (b) historical simulation of residuals (bootstrap). These pdfs of VaR show clearly that the VaR are statistically different for different distributional assumptions of the error term. Since the historical simulation shows significantly higher VaR it is preferable to use it rather than Normal distribution.

Figure 6 shows the CDS spreads and log-differences of CDS spreads. The CDS spreads for BAC and JPM seem to move together to some extent. As with equity volatility the CDS spreads were higher for JPM before the financial crisis and lower for the most time starting from the financial crisis. The log-differences of CDS spreads exhibit volatility clustering similar to equity returns. Figure 7 shows the leverage of BAC and JPM and the dynamics is similar to the CDS spreads with BAC leverage highly exceeding JPM leverage starting from the financial crisis.

The systemic risk measures of the marginal expected shortfall (MES), LRMES and SRISK over time are presented in Figures 8-10. All the graphs use posterior means of parameters of the DCC-GTARCH model and equations (3)-(6) for computation of the measures of interest. Half of the sample is used for MES of the first observation in 2006. We can see that the MES results also show higher risks for BAC starting from the crisis when BAC leverage increased dramatically and lower MES before the crisis. However, graphs are close and more careful analysis of the distributions of MES at a particular point is needed. Graphs of LRMES and SRISK show similar patterns with peaks during the financial crisis and potential treasury default with debt ceiling reached in August 2011. The SRISK average values presented in Figure 10 are similar to values reported by VLAB such as in Table 5. For example, at the end of the sample (2012/12/31) SRISK is about 104 $ billion for BAC and 75.4 $

billion for JPM using the the GJR-GARCH model as in Brownless and Engle (2012). The VLAB values are 101 \$ billion for BAC and 75.8 \$ billion for JPM.[4]

Finally we consider the whole posterior distribution for $MES_T, LRMES_T$ and $SRISK_T$ derived from the posterior distributions of $\sigma_T, \sigma_{m,T}, \rho_T$ obtained from the MCMC draws. Figure 11 shows the distribution of MES and SRISK for JPM at the end of the sample (T=2012/12/31) which is in the period of low volatility, while Figure 12 shows these measures in the period of high volatility (T=2008/08/29). We present the results when the GTARCH, GJR-GARCH and GARCH models are used. The interesting implication of the GTARCH model is that the results for volatility, MES and SRISK are lower in a period of low volatility and higher in a period of high volatility compared to GJR-GARCH and GARCH. GARCH model is less responsive than other two models to the periods of high and low volatility as it has no asymmetric news effect that captures risk-aversion. It seems that the TGARCH model captures risk-aversion better than GJR-GARCH model that is a commonly used model in the literature.[5]

For the remainder of the graphs we use the GTARCH model. Figures 13-15 compare the BAC and JPM posterior pdfs of MES, LRMES and SRISK for the low volatility time (T=2012/12/31). It turns out that their measures of risk are statistically different with distributions not crossing. This means that in the periods of low volatility the rankings of BAC being above JPM are justified distinguishing firms in terms of severity of the systemic risks they impose on the system. Figures 16-17 show MES and SRISK for JPM and BAC at the time of high volatility (T=2008/08/29) and we see that the distributions are close to each other with 95% highest posterior density intervals intersecting. JPM had higher leverage on that day and it resulted in somewhat higher SRISK but the results for BAC and JPM are not statistically significant. The results not presented here to save space indicate that the same pattern happens at other dates in periods of high volatility.

7 Conclusion

In this paper I considered Bayesian estimation of systemic risks. Using a new asymmetric GARCH model and capturing uncertainty around the measures I found that MES, LRMES and SRISK are statistically different for major financial firms at the times of low volatility, however, they may be very close at the times of uncertainty such as the financial crisis. This leads to policy implication that banks and other

[4]The results may the difference in estimation period used and constraints imposed on the GJR-GARCH model by the VLAB.

[5]The other asymmetric GARCH model is EGARCH

systemically important firms can not be taxed differently based on SRISK measure suggested by Acharya et. al (2010).

The paper has several contributions. This is the first paper to introduce Bayesian analysis for the systemic risk measures and derive the full distribution of those measures compared to simple point estimates used in the literature. Second, a new asymmetric GTARCH model introduced in this paper generalizes popular asymmetric volatility GJR-GARCH model and improves its properties. Third, I provide the whole distribution of systemic risk measures and show how to distinguish risks of different institutions. I also estimate GTARCH volatility of log-difference in CDS spreads showing alternative measures of financial risks.

For the future work I would like to consider different distributional assumptions for the error term. It would be also interesting to compare the market based measures of systemic risks used in this paper to the results of macroprudential stress tests.

References

Acharya, V., Engle, R. and Richardson, M. (2012). "Capital Shortfall: A New Approach to Ranking and Regulating Systemic Risks," *American Economic Review* 102 (3), 59-64.

Acharya, V., Engle, R., and Pierret, D. (2013). "Testing Macroprudential Stress Tests: The Risk of Regulatory Risk Weights," Working Paper, Department of Finance, New York University.

Acharya, V., Pedersen, L., Philippe, T., and Richardson, M. (2010). "Measuring systemic risk," Working Paper, Department of Finance, New York University.

Bisias, D., Flood, M., Lo, A. W., and Valavanis, S. (2012). "A survey of systemic risk analytics." Working paper #0001, Office of Financial Research.

Brownlees, C., and R. Engle, (2012) "Volatility, Correlation and Tails for Systemic Risk Measurement," Working Paper, Department of Finance, New York University.

Brunnermeier, M. K. and Oehmke, M. "Bubbles, financial crises, and systemic risk." Handbook of the Economics of Finance, Volume 2, (forthcoming).

Chib, S. and E. Greenberg (1994) "Bayes inference in regression models with ARMA(p, q) errors", Journal of Econometrics, 64, 183206.

Chib, S. and E. Greenberg (1995) "Understanding the Metropolis-Hastings Algorithm," the American Statistician, 49, 327335.

Glosten, L. R., Jagananthan, R., and Runkle, D. E. (1993). On the relation between the expected value and the volatility of the nominal excess return on stocks. The Journal of Finance, 48, 17791801.

Goldman, E. and Tsurumi, H. (2005). Bayesian Analysis of a Doubly Truncated ARMA-GARCH Model. Studies in Nonlinear Dynamics and Econometrics, 9 (2), article 5.

Hull, J.C., Predescu, M., White, A., 2004. The relationship between credit default swap spreads, bond yields, and credit rating announcements. Journal of Banking and Finance 28, 27892811.

Carr, P. and Wu, L. (2011). A Simple Robust Link Between American Puts and Credit Protection, Review of Financial Studies, 24(2), 473-505.

Nakatsuma, T., (2000). "Bayesian analysis of ARMA-GARCH models: A Markov chain sampling approach", Journal of Econometrics, 95, 5769.

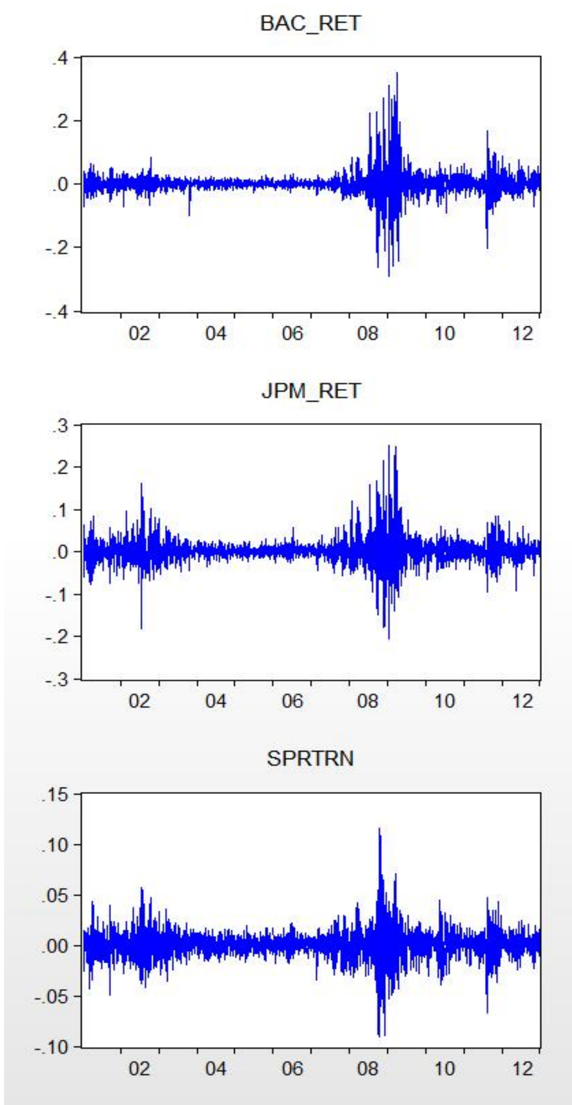

Figure 1: Returns: BAC, JPM, SPX

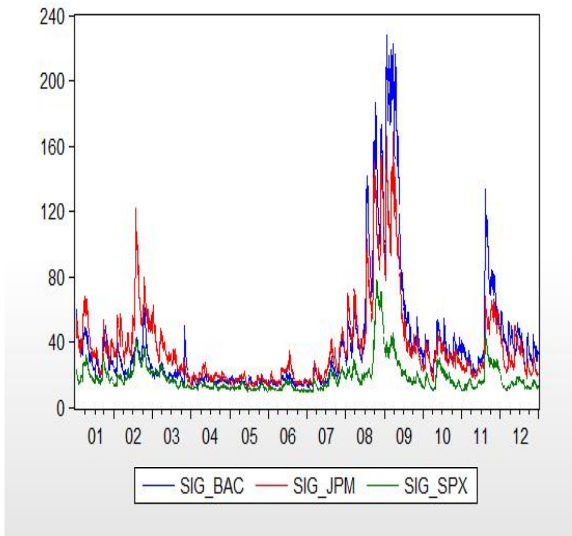

Figure 2: Annualized Volatility (GTARCH): BAC, JPM, SPX

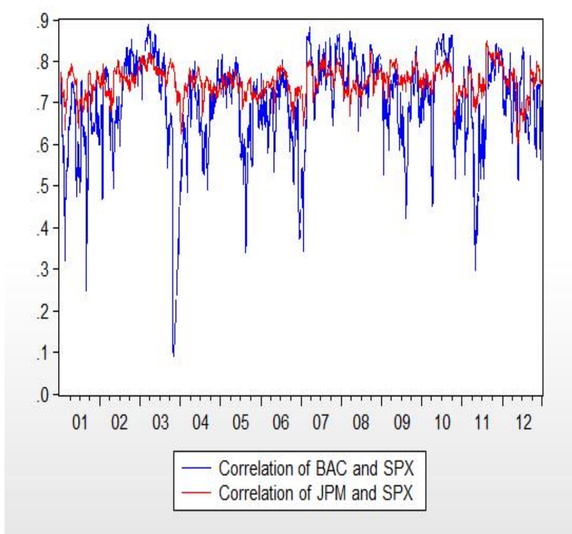

Figure 3: Dynamic correlation with the market (DCC-GTARCH): BAC, JPM

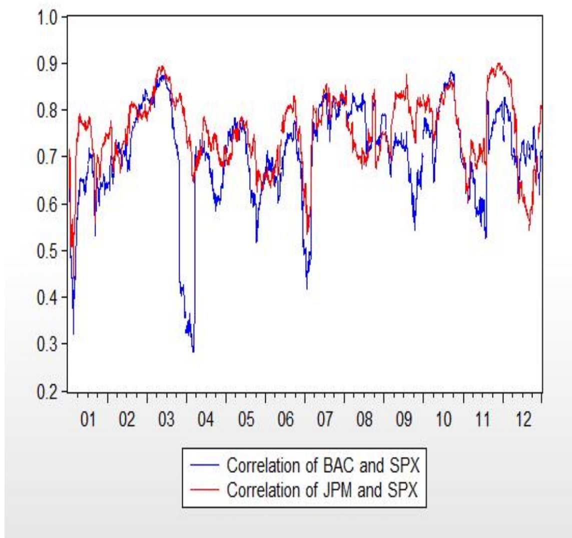

Figure 4: 100 day rolling correlation with the market : BAC, JPM

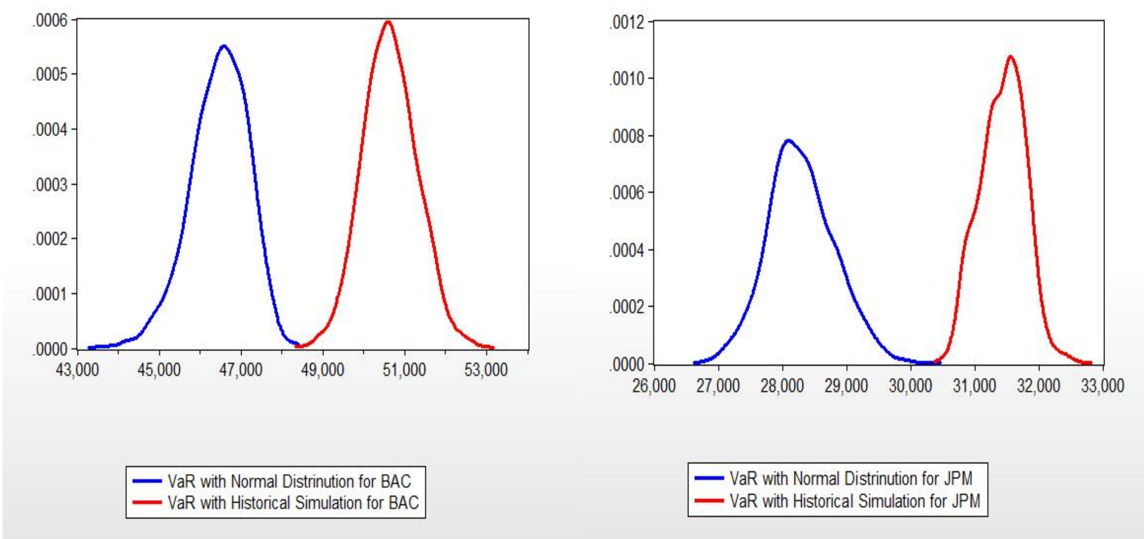

Figure 5: Distribution of 1% quantile (VaR) for the $1 million portfolio if (a) the residuals are normal and (b) corrected for fat tails: BAC, JPM

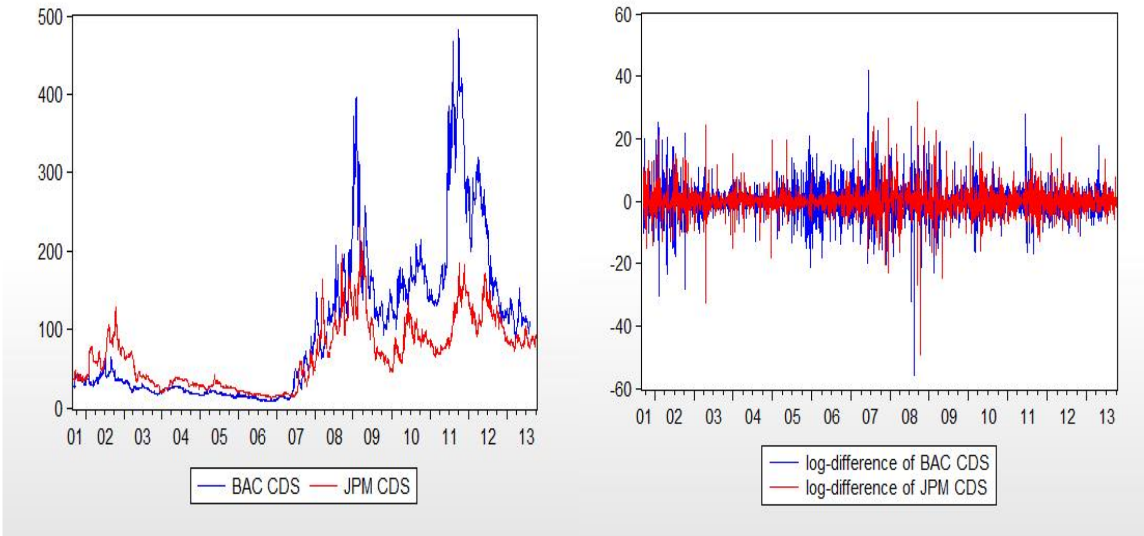

Figure 6: (a) CDS spreads and (b) log-difference of CDS spreads: BAC, JPM

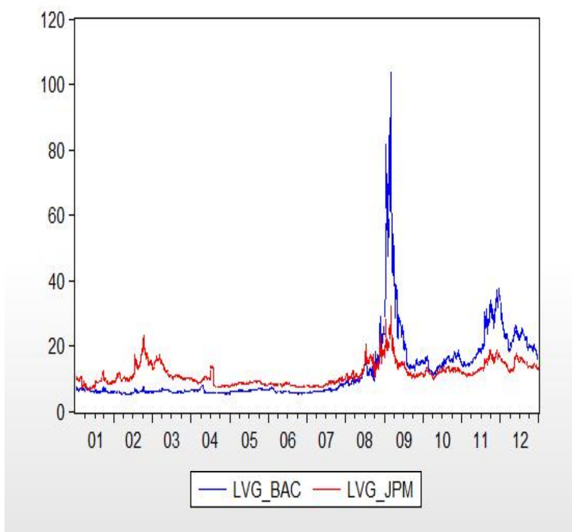

Figure 7: Leverage: BAC and JPM

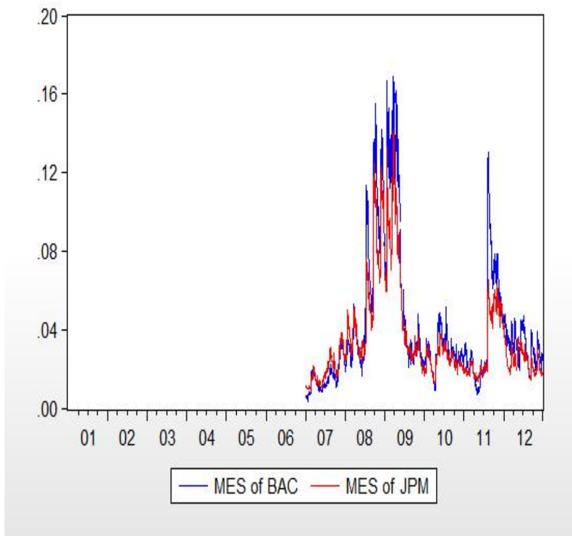

Figure 8: Marginal Expected Shortfall (MES): BAC, JPM

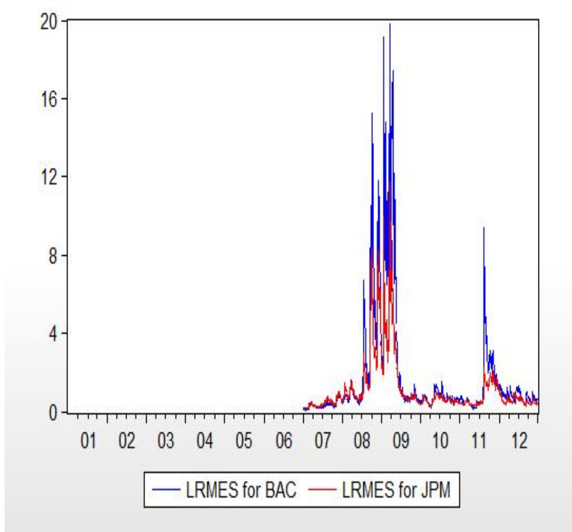

Figure 9: Long Run Marginal Expected Shortfall (LRMES): BAC, JPM

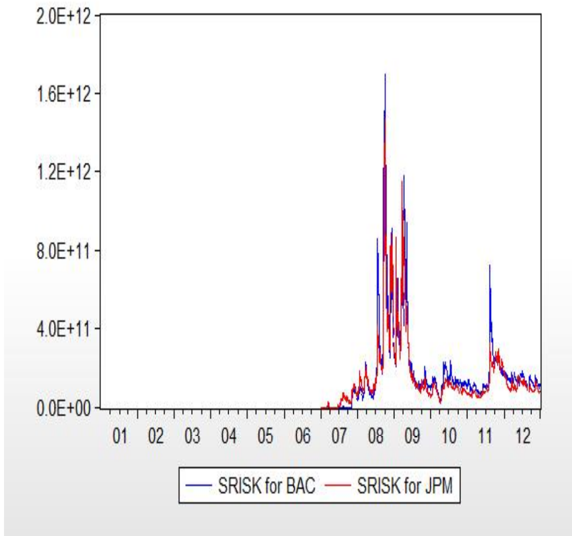

Figure 10: SRISK: BAC, JPM

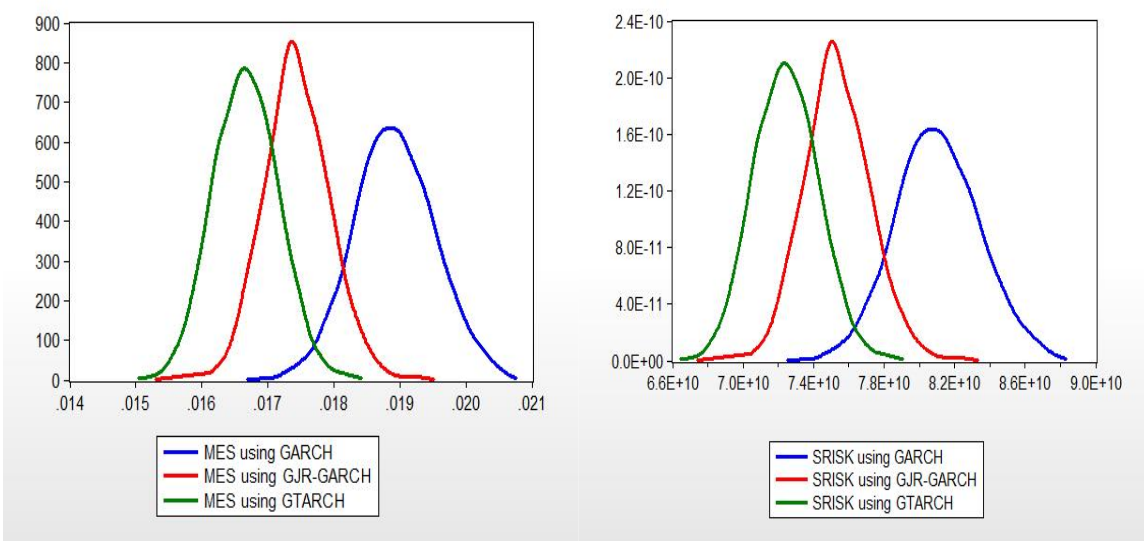

Figure 11: PDFs of MES and SRISK for different volatility models in the period of low volatility: JPM (2012/12/31)

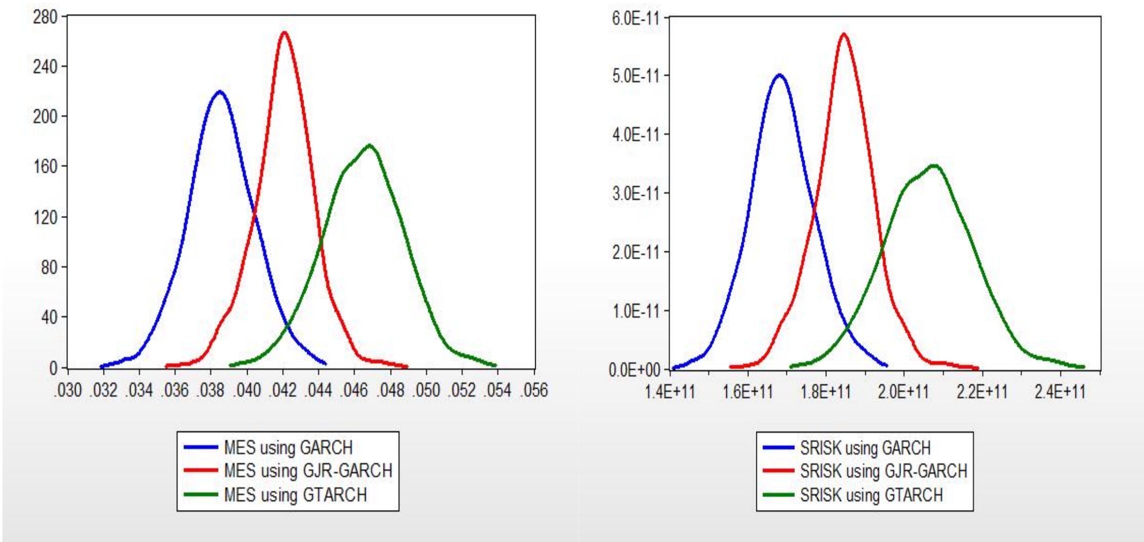

Figure 12: PDFs of MES and SRISK for different volatility models in the period of high volatility: JPM (2008/08/29)

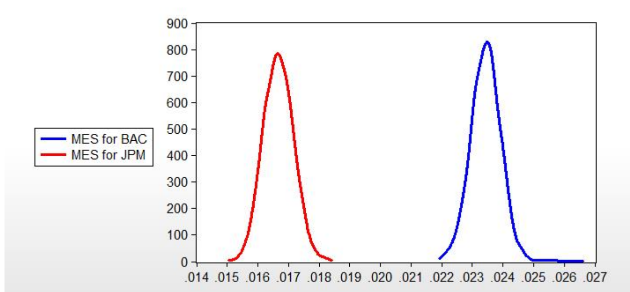

Figure 13: PDFs of Marginal Expected Shortfall in the period of low volatility: BAC, JPM (2012/12/31)

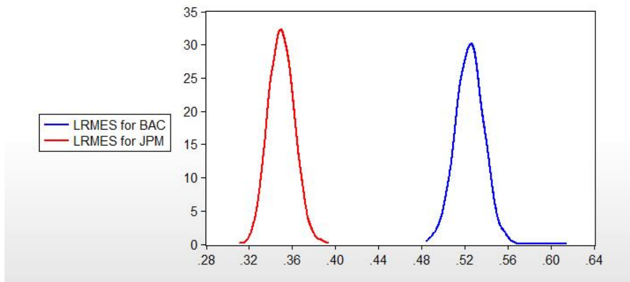

Figure 14: PDFs of Long Run Marginal Expected Shortfall in the period of low volatility: BAC, JPM (2012/12/31)

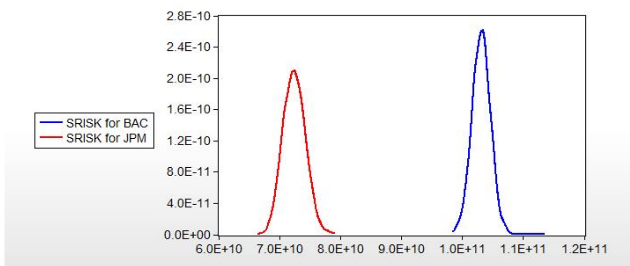

Figure 15: PDFs of SRISK in the period of low volatility: BAC, JPM (2012/12/31)

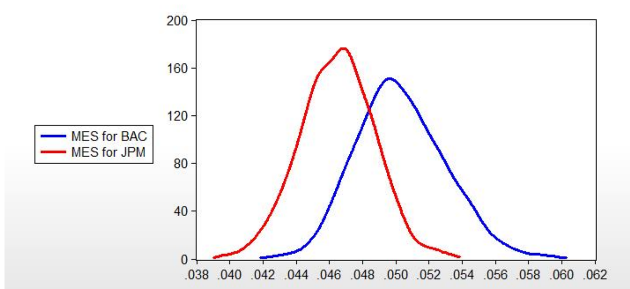

Figure 16: PDFs of Marginal Expected Shortfall in the period of high volatility: BAC, JPM (2008/08/29)

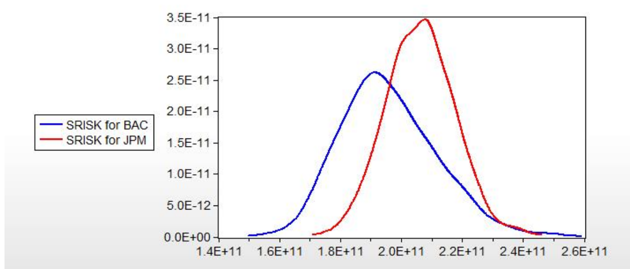

Figure 17: PDFs of SRISK in the period of high volatility: BAC, JPM (2008/08/29)