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Research Paper

Procyclicality mitigation for initial margin models with asymmetric volatility

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ABSTRACT

We apply a variety of volatility models in setting the initial margin requirements for central clearing counterparties (CCPs) and show how to mitigate procyclicality using a three-regime threshold autoregressive model. In order to evaluate the initial margin models, we introduce a loss function with two competing objectives: risk sensitivity and procyclicality mitigation. The trade-off parameter between these objectives can be selected by the regulator or CCP, depending on the specific preferences. We also explore the properties of asymmetric generalized autoregressive conditional heteroscedasticity (asymmetric GARCH) models in the threshold GARCH family, including the spline-generalized threshold GARCH model, which captures high-frequency return volatility and low-frequency macroeconomic volatility as well as an asymmetric response to past negative news in both past innovations (ARCH) and volatility (GARCH) terms. We find that the more general asymmetric volatility model has a better fit, greater persistence of negative news, a higher degree of risk aversion and an important effect on macroeconomic variables for the low-frequency volatility component of the Standard & Poor's 500 and S&P/Toronto Stock Exchange returns.

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1 INTRODUCTION

The mandatory use of clearing in certain markets is one of the cornerstone regulations introduced to prevent another global financial crisis. However, the rules implemented have not been tested in crisis conditions.

Central clearing counterparties (CCPs) base their risk management systems on a tiered default waterfall relying on two main types of resources provided by their members: margins ("defaulter pays") and default fund (mutualized) contributions. The Principles for Financial Market Infrastructures (PFMI), published by the Committee on Payment and Settlement Systems–Technical Committee of the International Organization of Securities Commissions (2012), require CCPs to hold enough capital to cover losses resulting from their two largest participants' failure and to set initial margins to cover at least 99% of potential future exposures.

The initial margins are typically set based on value-at-risk (VaR) calculations, as documented in, for example, Murphy *et al* (2014, 2016), Knott and Polenghi (2006) and Houllier and Murphy (2017). The VaRs in this literature are computed using conditional variances, such as the generalized autoregressive conditional heteroscedasticity (GARCH) and exponentially weighted moving average (EWMA) RiskMetrics models. These models capture the stylized facts of volatility clustering and provide a timely risk measure reflecting the most recent news. However, it is also important to account for the asymmetric response of volatility to negative news (risk aversion), changes in macroeconomic factors and long-run dynamics. Since VaRs move with the volatility, the properties of the underlying volatility models, such as risk aversion and forecasting, are essential for setting initial margin requirements.

As documented in Murphy *et al* (2014, 2016), Brunnermeier and Pedersen (2009) and Glasserman and Wu (2018), margin models are typically procyclical and may negatively impact members' funding liquidity in times of crisis. There is a need for margins to adjust to changes in the market and be responsive to risk. Thus, margins are higher in times of stress and lower when volatility is low. However, this practice may produce large changes in margins when markets are stressed, which in turn may lead to liquidity shocks. Brunnermeier and Pedersen (2009) showed that margins can be destabilizing, with stresses in market and funding liquidity leading to liquidity spirals. Biais *et al* (2016) studied moral hazard due to the excessively risky behavior of safer members when less procyclical margins are used; this risky behavior occurs because a particular member's default, resulting in a loss exceeding its margin (collateral) and default fund contributions, is covered by the mutualized default fund contributions of surviving members. Raykov (2018) further explores the conditions under which members are motivated to exit the CCP due to a lack of confidence in the CCP's stability and finds that some smoothing of margins may stimulate trading and restore confidence in the market. Cruz Lopez *et al* (2017) suggest setting margin requirements based on the CoMargin measure (defined as the VaR of a clearing member conditional on the financial distress of at least one other member), and thus taking into account the correlation of losses between the CCP's participants. They also note that correction for procyclicality is desirable for CoMargin.

Procyclicality in margin requirements is a concern for regulators with regard to the stability of the financial system. Under the PFMI, CCPs should adopt forward-looking and relatively stable margin requirements. CCPs try to reduce the procyclicality of their models by using various methods, including setting floors on the margin. Some of these methods are discussed in Murphy *et al* (2016). Article 28 of the European Market Infrastructure Regulation (EMIR) Regulatory Technical Standards (European Commission 2013) requires the use of at least one of the following three tools to create margin buffers and reduce procyclicality: setting a floor margin buffer of 25% or higher to be used in times of stressed conditions; assigning at least a 25% weight to stressed observations; and setting a floor based on the maximum volatility over a ten-year historical look-back period. In addition, Murphy *et al* (2016) suggest setting limits on how quickly the margins can be raised.

In this paper, we explore the procyclicality of initial margin requirements based on VaR volatility models. We suggest procyclicality can be reduced using a three-regime model rather than using ad hoc tools. Moreover, unlike other literature, we introduce not only the lower bound (floor) but also the upper bound (ceiling) for the initial margins, as the upper bound is essential at times of liquidity stress in the market. We apply a threshold autoregressive (TAR) model with three regimes (3TAR).

Finally, we define and use a loss function with different degrees of trade-off between two competing objectives of the CCP: risk sensitivity and mitigation of procyclicality. If the margins were allowed to be set within two moving thresholds and the high-volatility regime were not persistent, margins would be more stable. Such policy could also be useful to manage expectations at times of stressed liquidity.

This study also reviews GARCH models that can be applied to set margins for various risk factors. It is common to use risk factors to estimate covariance matrixes of portfolios. While we focus on equity indexes in this paper, the same models can be applied to other asset classes, such as oil futures, interest rate swap rates, credit default swap rates and currency forwards (see, for example, Murphy *et al* 2016; Knott and Polenghi 2006). We also demonstrate a flexible volatility model that can capture a high degree of risk aversion as well as the effects of macroeconomic variables.

Macroeconomic variables can be used for VaR model stress testing under various scenarios and improve forecasting, as shown in Engle *et al* (2013).

Since tail risk measures typically incorporate forecasts of volatility, model specification is important. GARCH and EWMA RiskMetrics models are popular with financial practitioners for measuring and forecasting volatility. Engle and Mezrich (1995) introduced a way to estimate VaR using a GARCH model, while Hull and White (1998) proved that a GARCH model performs better than a stochastic volatility model in the calculation of VaR. The Glosten–Jagannathan–Runkle-GARCH (GJR-GARCH) model (Glosten *et al* 1993) was used by Brownlees and Engle (2017), among others, for forecasting volatility and measurement of tail and systemic risks.

A typical feature of the GARCH family models is that the long-run volatility forecast converges to a constant level. An exception is the spline-GARCH model of Engle and Rangel (2008) that allows the unconditional variance to change with time as an exponential spline, and the high-frequency component to be represented by a unit GARCH process. This model may incorporate macroeconomic and financial variables into the slow-moving component and, as shown in Engle and Rangel (2008), improves long-run forecasts of international equity indexes. In this model, the unconditional volatility coincides with the low-frequency volatility.

The widely used asymmetric GJR-GARCH model has the problem that the unconstrained estimated coefficient of α often has a negative value for equity indexes. A typical solution to this problem is to set the coefficient of α to zero in the constrained maximum likelihood. Following Goldman (2017), we use a generalized threshold GARCH (GTARCH) model, where both coefficients (α and β) in the GARCH model are allowed to change to reflect the asymmetry of volatility due to negative shocks. We use data for the US and Canadian equity indexes, Standard & Poor's 500 (S&P 500, SPX) and S&P/Toronto Stock Exchange Composite (TSX) as well as a numerical example to estimate various asymmetric volatility models. We find that the most general GTARCH model fits better and does not have a negative alpha bias. We also find greater persistence and more risk aversion in the GTARCH models.

We add macroeconomic variables for gross domestic product (GDP) growth, inflation, overnight interest rate and exchange rate into the spline model for the slow-moving component. The spline-macro model results in a smaller number of optimal knots for SPX and has a better fit for both SPX and TSX.

The paper is organized as follows. Section 2 presents GTARCH and spline-GTARCH models and tail risks. In Section 3, we perform data analysis for the S&P 500 and S&P/TSX indexes, while in Section 4 we compare tail risks and perform backtests of all models. Next, we analyze procyclicality properties and estimate a three-regime TAR model for setting a floor and a ceiling on margins as well as speed limits. Section 5 presents our conclusions and discusses further work. Details of maximum likelihood estimation, Monte Carlo simulations showing negative bias of α in the GJR-GARCH model for unconstrained optimization, and additional results are presented in the online appendix.

2 ASYMMETRIC THRESHOLD GENERALIZED AUTOREGRESSIVE CONDITIONAL HETEROSCEDASTICITY MODELS

In this section, we present the GTARCH model and a subset family of models including GJR-GARCH, GTARCH0 and GARCH. Next, we add a spline to the GTARCH model, extending the analysis of Engle and Rangel (2008).

2.1 The GTARCH model

One of the stylized facts in empirical asset pricing is the negative correlation between asset returns and volatility, commonly explained by risk aversion and the leverage effect. In a popular threshold ARCH or GJR-GARCH model (Glosten *et al* 1993), a negative return results in an asymmetrically higher effect on the next-day conditional variance compared with a positive return.

Consider a time series of logarithmic returns r_t with constant mean μ and GJR-GARCH conditional variance σ_t^2 given by

$$\left.\begin{aligned} r_t &= \mu + u_t = \mu + \sigma_t \varepsilon_t, \\ \sigma_t^2 &= \omega + \alpha u_t^2 + \gamma u_t^2 I(r_{t-1} - \mu < 0) + \beta \sigma_{t-1}^2, \end{aligned}\right\} \tag{2.1}$$

where ε_t are Gaussian (or other distribution) independent random variables with mean zero and unit variance, $I(r_{t-1} - \mu < 0)$ is a dummy variable equal to 1 when the previous-day innovation u_{t-1} is negative, α and β are GARCH parameters and γ is an asymmetric term capturing risk aversion. The stationarity condition for the GJR-GARCH model is given by $1 - \alpha - \beta - \frac{1}{2}\gamma > 0$.

However, there is a problem with the threshold ARCH model above, since coefficient α may take negative values in practice. In such a case, a constrained optimization imposing positivity on all variance parameters results in $\alpha = 0$. Goldman (2017) suggested using a more general threshold GARCH (GTARCH) model:

$$\sigma_t^2 = \omega + \alpha u_t^2 + \gamma u_t^2 I(r_{t-1} - \mu < 0) + \beta \sigma_{t-1}^2 + \delta \sigma_{t-1}^2 I(r_{t-1} - \mu < 0), \tag{2.2}$$

where the additional coefficient δ reflects the degree of asymmetric response in the GARCH term. In this model, both parameters γ and δ establish the asymmetric response of volatility to negative shocks. The results below show that allowing both ARCH and GARCH parameters to change with negative news results in a better statistical fit and smaller information criteria. Moreover, the GTARCH model not only better captures the leverage effect but also shows greater persistence for negative returns compared with its subset GJR-GARCH model. In addition, the coefficients of μ and ω could be allowed to change with the regime of negative news to make the model even more flexible. The GTARCH is a generalized model with the following subset of models: GJR-GARCH ($\delta = 0$), GTARCH0 ($\gamma = 0$) and GARCH ($\gamma = 0$ and $\delta = 0$).

The stationarity condition for the GTARCH model is given by $1 - \alpha - \beta - \frac{1}{2}\gamma - \frac{1}{2}\delta > 0$. The more general GTARCH model shows different dynamics for GARCH parameters when the news is negative, due to the flexibility of its parameters, and allows for greater persistence in the regime of negative news. This in turn removes the negative bias from α, which measures the reaction to the positive news. At the same time, the estimation of the extra parameter δ in the model using the maximum likelihood is a straightforward extension, as shown in the online appendix.

In addition to GTARCH models, we estimate the EWMA model, defined as

$$\sigma_t^2 = (1 - \lambda) r_{t-1}^2 + \lambda \sigma_{t-1}^2, \tag{2.3}$$

where λ is a smoothing parameter estimated using the maximum likelihood. This model is not stable but is a benchmark for one-day volatility forecasts, with a typical estimate of $\lambda = 0.94$ frequently used in the industry. The EWMA model is popular for measuring tail risks, as will be discussed below.

Finally, there has been a growing literature on the use of intraday measures of variance, such as realized variance, computed as the sum of squared returns using five-minute intervals. Andersen *et al* (2003) showed that the autoregressive fractionally integrated moving average model can be used for forecasting realized variance. Recent contributions on using realized volatility (RV) measures for predicting future variance include the heterogeneous autoregressive model of Corsi (2009) and the model using VIX by Bekaert and Hoerova (2014). Following this literature, we evaluate each model in this paper using RV as a benchmark observable variance.

2.2 The spline generalized threshold GARCH (spline-GTARCH) model

There is a growing literature incorporating economic variables for modeling and forecasting financial volatility. For example, Officer (1973), Schwert (1989), Roll (1988), Balduzzi *et al* (2001) and Andersen *et al* (2007) found that, even though the linkages between aggregate volatility and economy are weak, volatility is higher during recessions and post-recessionary stages and lower during normal periods. Engle and Rangel (2008) introduced the spline-GARCH model, which combines high-frequency financial returns and low-frequency macroeconomic variables. They analyze the effects of macroeconomic variables on the slow-moving component of volatility using a spline. This model loosens the assumption of volatility mean reversion to a constant level, which is a property of a stable GARCH model. Instead, the long-run unconditional variance is dynamic.

Combining the Engle and Rangel (2008) spline-GARCH model with the general GTARCH asymmetric volatility model in (2.2), we get

$$\begin{aligned} r_t &= \mu + \sqrt{\tau_t \sigma_t^2} z_t, \\ \sigma_t^2 &= \omega + \alpha\left(\frac{(r_{t-1}-\mu)^2}{\tau_{t-1}}\right) + \gamma\left(\frac{(r_{t-1}-\mu)^2}{\tau_{t-1}}\right) I(r_{t-1}-\mu<0) \\ &\qquad + \beta\sigma_{t-1}^2 + \delta\sigma_{t-1}^2 I(r_{t-1}-\mu<0), \\ \tau_t &= c\exp\left(\sum_{i=1}^{k} w_i((t-t_{i-1})_+)^2 + m_t\gamma\right), \\ &\qquad (t-t_i)_+ = \begin{cases} (t-t_i) & \text{if } t \geqslant t_i, \\ 0 & \text{otherwise,} \end{cases} \end{aligned} \tag{2.4}$$

where z_t is a standard Gaussian white noise process, σ_t^2 is a GTARCH process with an unconditional mean of 1, m_t is the set of weakly exogenous variables (ie, macroeconomic variables) and $(t_0 = 0,\ t_1, t_2, \dots, t_k = T)$ is a partition of the total number of observations T into k equal subintervals. The constant term in the GTARCH equation is $\omega = (1-\alpha-\beta-\frac{1}{2}\gamma-\frac{1}{2}\delta)$, and $\omega > 0$ if the GTARCH process is stable. Since the constant term in the GARCH variance equation is normalized, the long-run (unconditional) variance is determined by the spline. A higher number of knots (k) implies more cycles in the low-frequency volatility, while parameters $w_1, \dots, w_k$ represent the sharpness of the cycles.

In (2.4), we simplified the return process with a constant μ instead of the time-variant conditional mean (which could be easily extended for a different process). In practice, we also dropped the constant w_0 in the quadratic spline, as it was never significant.[1] The maximum likelihood estimation (MLE) for joint estimation of parameters in the spline-GTARCH model is presented in the online appendix.

Both in-sample and out-of-sample daily tail risks can be computed based on the volatility model used for estimating and forecasting of portfolio returns. The VaR is typically computed using either a parametric assumption for the distribution of returns or bootstrapped standardized residuals (also called “filtered historical simulation”, based on the Hull and White (1998) method). If the standardized residuals

[1] A similar spline-GARCH specification with constant μ and $w_0 = 0$ is used by the NYU Stern Volatility Laboratory (V-Lab) (see http://vlab.stern.nyu.edu).

$e_t = (r_t - \mu)/\sigma_t$ still have fat tails after adjustment for time-varying volatility, the Hull–White method is often used.

3 DATA ANALYSIS

In this section, we perform data analysis for the S&P 500 (SPX) and S&P/TSX (TSX). The results illustrating estimation and negative bias of α in the unconstrained GJR-GARCH model and Monte Carlo experiments for the GTARCH model are shown in the online appendix (Table A1).

The daily SPX data for the period between October 8, 2002 and December 30, 2016 was obtained from the Center for Research in Security Prices (CRSP) in the Wharton Database, while the TSX data for the period between March 17, 2003 and March 31, 2017 was obtained from Bloomberg. For both series, we found logarithmic returns that resulted in 3500 observations. Realized variances computed using five-minute returns were obtained from the Oxford-Man Institute of Quantitative Finance.[2]

For the spline model with macroeconomic variables, we used similar data to Engle and Rangel (2008), including quarterly nominal GDP growth rates for both countries, the daily US federal funds effective rate and Canadian overnight money market financing rate, monthly consumer price index inflation for both countries, the daily trade-weighted US dollar index and US-dollar-to-Canadian dollar (USD/CAD) exchange rates. We also added monthly unemployment rates for each country. Table A2 in the online appendix provides the description and data sources for all variables. It also explains how we transformed macroeconomic variables.

Tables 1 and 2 show the results of the estimated GTARCH family models for SPX and TSX, respectively. Based on the Bayesian information criterion (BIC) with a heavier penalty for extra parameters, the GTARCH model without spline is preferred, while, using the Akaike information criterion (AIC), the spline-macro-GTARCH is the superior model. Note that both selected models include the most general GTARCH specification with the presence of asymmetry in both ARCH and GARCH terms. Moreover, the asymmetric term δ goes up to 0.24 in the SPX spline-GTARCH model, making the response to negative news even more asymmetric than GTARCH without spline with $\delta = 0.16$. The optimal number of knots in the SPX spline model is 17, while the number of knots goes down to 8 when we add macroeconomic variables. Macroeconomic variables are useful in modeling the low-frequency component, as their presence reduces the number of knots for cycles and they have a statistically significant effect on long-run volatility dynamics. We model

[2] See http://realized.oxford-man.ox.ac.uk/. The number of observations for the RV was slightly smaller than that for daily returns: 3487 observations for SPX and 3480 observations for TSX.

macroeconomic variables for daily volatility forecasting in the slow-moving component. Thus, statistically significant macroeconomic variables in the low-frequency component could be used for the stress testing of VaRs, which is a typical regulatory requirement. The following variables are statistically significant at 10% for predicting the low-frequency volatility component for SPX:

- interest rate (IR) and interest rate volatility (IR_V), both of which have a positive effect on SPX volatility;
- volatility of the unemployment rate (unemp_V), which has a negative effect on SPX volatility;
- volatility of USD trade-weighted index (USD_V), which has a positive effect on SPX volatility;
- GDP growth, which has a negative effect on SPX volatility.

All the signs are as expected, except for the volatility of the unemployment rate. It might be the case that the reduction (rather than increase) in unemployment rate is driving these results.

The spline-macro (SMacro) model has lower persistence than the spline and no-spline models. Spline-macro models thus have a faster convergence of variance to the long-run spline macroeconomic component because the long-run component is not as smooth as in the simple spline model.

In addition to the volatility models presented in Tables 1 and 2, we estimated the RiskMetrics EWMA volatility model that is commonly used as a benchmark in volatility forecasting and VaR estimation. The MLE for the EWMA model resulted in the following smoothing parameters, with standard error given in brackets and information criteria:

$$\begin{aligned}
&\text{SPX:} \quad \lambda = 0.9409\ (0.0049), \quad \text{AIC} = 2.7262, \quad \text{BIC} = 2.7279,\\
&\text{TSX:} \quad \lambda = 0.9369\ (0.0055), \quad \text{AIC} = 2.8222, \quad \text{BIC} = 2.8240.
\end{aligned}$$

In both cases, the smoothing parameter is very close to 0.94, which is frequently used in practice. Thus, the US and Canadian indexes have similar EWMA volatility dynamics. In order to evaluate how parameters change over time, in Table 3(a) we present the results of the GTARCH and EWMA models for SPX data for three subsamples: before the 2008–9 crisis (October 8, 2002 to December 31, 2007; 1278 observations), before and including the crisis (October 8, 2002 to December 31, 2009; 1772 observations) and postcrisis (January 1, 2010 to December 30, 2016; 1773 observations). While the estimates for the different periods are within two standard deviations of each other, the smoothing parameter for the EWMA model

TABLE 1 Estimation results for GTARCH, spline-GTARCH and spline-macro-GTARCH models: SPX. [Table continues on next three pages.]

	GTARCH						GTARCH0					
	SMacro		Spline		No spline		SMacro		Spline		No spline	
Parameter	Parm	SD	Parm	SD	Parm	SD	Parm	SD	Parm	SD	Parm	SD
μ	0.013	(0.015)	0.008	(0.015)	0.000	(0.000)	−0.726	(0.154)	0.026	(0.014)	0.001	(0.000)
ω					0.023	(0.004)					0.022	(0.003)
α	0.000	(0.023)	0.000	(0.022)	0.000	(0.013)	0.061	(0.018)	0.065	(0.006)	0.078	(0.008)
β	0.781	(0.030)	0.771	(0.028)	0.837	(0.019)	0.729	(0.033)	0.734	(0.015)	0.789	(0.015)
γ	0.122	(0.023)	0.131	(0.025)								
δ	0.230	(0.035)	0.243	(0.033)	0.160	(0.025)	0.316	(0.031)	0.310	(0.025)	0.249	(0.025)
c	0.936	(0.289)	2.718	(0.222)			0.846	(0.513)	1.800	(0.300)		
w_1	−0.711	(0.118)	−1.973	(0.236)			−0.726	(0.154)	−1.454	(0.370)		
w_2	0.854	(0.251)	4.110	(0.626)			0.821	(0.448)	2.860	(1.030)		
w_3	2.106	(0.567)	−2.430	(0.712)			2.365	(0.838)	−1.300	(1.157)		
w_4	−4.443	(0.743)	−0.008	(0.700)			−4.796	(0.936)	−0.894	(0.900)		
w_5	2.831	(0.455)	1.669	(0.679)			2.918	(0.468)	2.948	(0.872)		
w_6	−1.002	(0.398)	−1.607	(0.687)			−0.872	(0.569)	−3.167	(1.101)		
w_7	0.736	(0.527)	1.628	(1.152)			0.651	(0.791)	3.230	(1.157)		
w_8	−0.859	(0.728)	−5.859	(1.744)			−1.153	(1.161)	−7.984	(1.153)		
w_9			6.568	(1.585)					8.707	(0.996)		
w_{10}			−1.421	(0.907)					−2.060	(0.763)		

TABLE 1 Continued.

	GTARCH						GTARCH0					
	SMacro		Spline		No spline		SMacro		Spline		No spline	
Parameter	**Parm**	**SD**	**Parm**	**SD**	**Parm**	**SD**	**Parm**	**SD**	**Parm**	**SD**	**Parm**	**SD**
w_{11}			−1.728	(0.815)					−2.658	(0.803)		
w_{12}			1.752	(0.815)					2.842	(0.783)		
w_{13}			−1.128	(0.782)					−1.278	(1.194)		
w_{14}			0.175	(0.796)					−0.011	(1.153)		
w_{15}			2.089	(1.145)					1.689	(1.088)		
w_{16}			−3.193	(1.194)					−3.076	(1.212)		
w_{17}			0.005	(1.105)					0.523	(1.220)		
Inflation	0.084	(0.109)					0.081	(0.150)				
Inflation_V	−1.309	(0.829)					−1.983	(0.748)				
IR	0.675	(0.152)					0.749	(0.165)				
IR_V	2.077	(1.166)					2.050	(2.079)				
unemp_V	−1.456	(0.779)					−5.099	(1.911)				
USD_V	0.992	(0.332)					1.176	(0.329)				
GDP	−0.218	(0.088)					−0.250	(0.154)				
GDP_V	0.224	(0.216)					0.500	(0.247)				
Persistence	0.957		0.958		0.987		0.948		0.953		0.991	
BIC	2.330		2.333		2.324		2.350		2.357		2.344	
AIC	2.291		2.292		2.313		2.313		2.319		2.336	

TABLE 1 Continued.

	GJR-GARCH						GARCH					
	SMacro		Spline		No spline		SMacro		Spline		No spline	
Parameter	**Parm**	**SD**	**Parm**	**SD**	**Parm**	**SD**	**Parm**	**SD**	**Parm**	**SD**	**Parm**	**SD**
μ	0.028	(0.013)	0.025	(0.012)	0.018	(0.013)	0.058	(0.013)	0.058	(0.013)	0.055	(0.014)
ω					0.023	(0.003)						
α	0.000	(0.000)	0.000	(0.000)	0.000	(0.019)	0.089	(0.010)	0.097	(0.009)	0.102	(0.011)
β	0.841	(0.016)	0.840	(0.013)	0.888	(0.019)	0.826	(0.018)	0.833	(0.015)	0.875	(0.012)
γ	0.177	(0.020)	0.189	(0.017)	0.175	(0.021)						
δ	0.249	(0.025)										
c	0.754	(0.193)	2.246	(0.418)			0.640	(0.158)	1.873	(0.373)		
w_1	−0.760	(0.106)	−2.215	(0.426)			−0.753	(0.107)	−1.994	(0.462)		
w_2	1.032	(0.241)	4.587	(1.211)			0.891	(0.222)	4.196	(1.281)		
w_3	1.854	(0.618)	−2.706	(1.591)			2.301	(0.566)	−2.589	(1.516)		
w_4	−4.267	(0.804)	0.185	(1.765)			−4.759	(0.759)	0.099	(1.424)		
w_5	2.752	(0.486)	1.645	(1.704)			2.917	(0.464)	2.109	(1.436)		
w_6	−0.972	(0.380)	−2.343	(1.519)			−0.964	(0.353)	−2.689	(1.521)		
w_7	0.837	(0.405)	3.429	(1.782)			0.911	(0.365)	3.071	(1.652)		
w_8	−1.130	(0.559)	−9.085	(2.101)			−1.532	(0.506)	−8.526	(2.104)		
w_9					10.785	(1.950)					10.411	(2.108)
w_{10}					−5.566	(1.875)					−4.571	(1.650)

TABLE 1 Continued.

	GJR-GARCH						GARCH					
	SMacro		Spline		No spline		SMacro		Spline		No spline	
Parameter	**Parm**	**SD**	**Parm**	**SD**	**Parm**	**SD**	**Parm**	**SD**	**Parm**	**SD**	**Parm**	**SD**
w_{11}					2.000	(2.214)					0.363	(1.501)
w_{12}					−1.081	(2.132)					−0.282	(1.476)
w_{13}					0.239	(1.876)					0.904	(1.412)
w_{14}					−0.013	(1.834)					−0.960	(1.667)
w_{15}					1.971	(2.014)					2.481	(1.911)
w_{16}					−3.218	(2.222)					−4.310	(1.951)
w_{17}					−0.201	(2.488)					1.848	(2.378)
Inflation	0.112	(0.103)					0.062	(0.092)				
Inflation$_V$	−1.190	(1.150)					−1.983	(1.122)				
IR	0.666	(0.152)					0.768	(0.149)				
IR$_V$	4.326	(2.033)					4.972	(1.541)				
unemp$_V$	−5.132	(4.081)					−6.893	(3.957)				
USD$_V$	1.028	(0.315)					1.167	(0.305)				
GDP	−0.245	(0.087)					−0.269	(0.076)				
GDP$_V$	0.403	(0.235)					0.601	(0.231)				
Persistence	0.929		0.935		0.975		0.915		0.930		0.977	
BIC	2.343		2.346		2.335		2.382		2.390		2.371	
AIC	2.308		2.309		2.335		2.347		2.353		2.326	

This table presents the results of all volatility models for SPX. We used SPX data for the period between October 8, 2002 and December 30, 2016. The sample size is 3500 observations. Only results with positivity constraints are reported. In addition to the volatility models presented in the table, we used the EWMA model, which resulted in a smoothing parameter estimate (Parm) and standard error (SD), given in parentheses: $\lambda = 0.9409$ (0.0049) and information criteria: AIC = 2.7262, BIC = 2.7279.

TABLE 2 Estimation results for GTARCH, spline-GTARCH and spline-macro-GTARCH models: TSX. [Table continues on next three pages.]

	GTARCH						GTARCH0					
	SMacro		Spline		No spline		SMacro		Spline		No spline	
Parameter	**Parm**	**SD**	**Parm**	**SD**	**Parm**	**SD**	**Parm**	**SD**	**Parm**	**SD**	**Parm**	**SD**
μ	0.013	(0.006)	0.010	(0.005)	0.000	(0.000)	−1.256	(0.772)	0.016	(0.005)	0.000	(0.000)
ω					0.002	(0.000)						
α	0.000	(0.062)	0.000	(0.011)	0.019	(0.009)	0.045	(0.010)	0.046	(0.005)	0.058	(0.007)
β	0.826	(0.058)	0.841	(0.014)	0.869	(0.013)	0.796	(0.023)	0.808	(0.020)	0.844	(0.013)
γ	0.079	(0.047)	0.076	(0.016)	0.069	(0.015)						
δ	0.168	(0.031)	0.192	(0.022)	0.137	(0.024)	0.227	(0.045)	0.239	(0.026)	0.194	**(0.000)**
c	1.334	(0.362)	0.871	(0.122)			1.314	(1.305)	0.713	(0.095)		
w_1	−1.332	(0.218)	−0.674	(0.140)			−1.256	(0.772)	−0.632	(0.218)		
w_2	3.000	(0.437)	2.026	(0.314)			2.911	(1.450)	1.938	(0.636)		
w_3	−0.468	(0.492)	−1.882	(0.498)			−0.408	(1.480)	−1.692	(0.681)		
w_4	−4.362	(0.980)	0.370	(0.985)			−4.445	(6.642)	0.235	(0.544)		
w_5	5.727	(1.627)	0.738	(1.138)			5.771	(10.502)	0.605	(0.772)		
w_6	−1.189	(1.090)	0.564	(0.773)			−1.575	(4.702)	0.474	(1.171)		
w_7	−6.600	(1.753)	−5.860	(1.009)			−5.827	(3.282)	−5.225	(1.311)		
w_8	8.539	(1.679)	8.342	(1.637)			7.997	(1.943)	7.426	(1.262)		
w_9	−3.104	(0.968)	−4.116	(1.457)			−2.804	(3.835)	−3.014	(1.217)		
w_{10}	−1.983	(0.993)	−0.486	(1.081)			−2.405	(4.409)	−1.745	(0.998)		

TABLE 2 Continued.

	GTARCH						GTARCH0					
	SMacro		Spline		No spline		SMacro		Spline		No spline	
Parameter	**Parm**	**SD**	**Parm**	**SD**	**Parm**	**SD**	**Parm**	**SD**	**Parm**	**SD**	**Parm**	**SD**
w_{11}	3.062	(1.029)	2.021	(0.868)			3.600	(3.912)	3.212	(1.035)		
w_{12}	−2.623	(0.880)	−1.660	(0.744)			−2.871	(2.801)	−2.686	(1.083)		
w_{13}	4.839	(1.106)	1.716	(0.739)			4.603	(3.546)	3.417	(0.950)		
w_{14}	−7.824	(1.689)	−1.846	(0.908)			−7.622	(4.738)	−5.281	(0.866)		
w_{15}	8.409	(2.383)	0.286	(0.999)			8.602	(6.186)	4.938	(1.715)		
w_{16}												
w_{17}												
Inflation	−0.191	(0.145)					−0.281	(0.138)				
Inflation_V	−10.000	(2.537)					−9.358	(19.123)				
IR	0.163	(0.140)					0.119	(1.013)				
IR_V	−2.888	(1.116)					−2.279	(6.446)				
unemp_V	1.604	(1.196)					0.853	(2.361)				
USDCAD_V	0.412	(0.143)					0.417	(0.148)				
GDP	−0.036	(0.054)					−0.036	(0.063)				
GDP_V	0.057	(0.059)					0.048	(0.240)				
Persistence	0.949		0.974		0.991		0.955		0.974		0.999	
BIC	2.326		2.313		2.306		2.337		2.324		2.312	
AIC	2.275		2.276		2.296		2.287		2.289		2.303	

TABLE 2 Continued.

Parameter	GJR-GARCH SMacro Parm	GJR-GARCH SMacro SD	GJR-GARCH Spline Parm	GJR-GARCH Spline SD	GJR-GARCH No spline Parm	GJR-GARCH No spline SD	GARCH SMacro Parm	GARCH SMacro SD	GARCH Spline Parm	GARCH Spline SD	GARCH No spline Parm	GARCH No spline SD
μ	0.016	(0.005)	0.015	(0.005)	0.014	(0.005)	0.026	(0.005)	0.025	(0.005)	0.024	(0.005)
ω					0.003	(0.001)					0.002	(0.001)
α	0.000	(0.000)	0.000	(0.000)	0.010	(0.009)	0.077	(0.008)	0.099	(0.010)	0.085	(0.010)
β	0.861	(0.015)	0.872	(0.013)	0.914	(0.011)	0.838	(0.017)	0.890	(0.011)	0.902	(0.012)
γ	0.122	(0.013)	0.130	(0.016)	0.107	(0.015)						
δ												
c	0.530	(0.113)	0.466	(0.063)			0.736	(0.038)	0.962	(0.094)		
w_1	−0.805	(0.330)	−0.286	(0.252)			−0.783	(0.241)	−0.418	(0.506)		
w_2	1.466	(0.854)	0.722	(0.798)			1.308	(0.669)	1.117	(1.509)		
w_3	1.060	(1.104)	−0.029	(1.165)			1.662	(0.863)	−0.373	(1.732)		
w_4	−5.151	(1.582)	−1.166	(1.343)			−5.982	(1.057)	−0.668	(1.844)		
w_5	6.368	(1.716)	1.635	(1.373)			6.999	(1.298)	0.440	(2.174)		
w_6	−0.871	(1.125)	0.173	(1.283)			−2.058	(1.531)	1.001	(1.863)		
w_7	−8.453	(2.149)	−5.863	(1.389)			−6.957	(2.570)	−5.400	(1.665)		
w_8	10.000	(1.689)	8.688	(1.551)			9.785	(2.378)	7.829	(1.922)		
w_9	−3.269	(1.066)	−4.278	(1.481)			−3.972	(1.353)	−3.860	(1.928)		
w_{10}	−2.301	(1.086)	−1.089	(1.386)			−1.900	(0.982)	−1.057	(1.910)		

TABLE 2 Continued.

	GJR-GARCH						GARCH					
	SMacro		Spline		No spline		SMacro		Spline		No spline	
Parameter	**Parm**	**SD**	**Parm**	**SD**	**Parm**	**SD**	**Parm**	**SD**	**Parm**	**SD**	**Parm**	**SD**
w_{11}	3.018	(1.252)	2.575	(1.351)			3.009	(1.007)	2.359	(2.149)		
w_{12}	−1.785	(1.251)	−1.462	(1.340)			−1.866	(1.072)	−1.136	(2.138)		
w_{13}	3.933	(1.325)	2.481	(1.643)			4.095	(1.268)	1.975	(2.456)		
w_{14}	−7.644	(1.583)	−5.155	(1.943)			−8.343	(1.480)	−4.909	(2.715)		
w_{15}	8.482	(2.208)	4.835	(2.432)			10.000	(1.696)	5.256	(3.321)		
w_{16}												
w_{17}												
Inflation	−0.263	(0.125)					−0.395	(0.105)				
Inflation$_V$	−9.413	(3.119)					−10.000	(1.798)				
IR	0.332	(0.140)					0.191	(0.091)				
IR$_V$	−0.806	(1.012)					−6.055	(1.211)				
unemp$_V$	1.197	(1.172)					5.960	(4.520)				
USDCAD$_V$	0.501	(0.137)					0.578	(0.107)				
GDP	−0.023	(0.040)					−0.032	(0.034)				
GDP$_V$	0.028	(0.044)					−0.022	(0.060)				
Persistence	0.922		0.938		0.978		0.915		0.989		0.987	
BIC	2.332		2.321		2.312		2.354		2.373		2.327	
AIC	2.284		2.288		2.303		2.307		2.340		2.320	

This table presents the results of all volatility models for TSX. TSX data is given for the period between March 17, 2003 and March 31, 2017. The sample size is 3500 observations. Only results with positivity constraints are reported. In addition to the volatility models presented in the table, we used the EWMA model, which resulted in a smoothing parameter estimate (Parm) and standard error (SD) given in parentheses: $\lambda = 0.9369$ (0.0055) and information criteria: AIC = 2.8222, BIC = 2.8240.

TABLE 3 Sensitivity results for GTARCH and EWMA: SPX.

(a) SPX results for three subsamples

	GTARCH						EWMA
	μ	ω	α	β	γ	δ	λ
2002–7	0.01	0.01	0.01	0.89	0.08	0.11	0.96
	(0.02)	(0.00)	(0.00)	(0.02)	(0.02)	(0.04)	(0.01)
2002–9	0.00	0.01	0.03	0.90	0.09	0.09	0.95
	(0.00)	(0.00)	(0.00)	(0.04)	(0.03)	(0.04)	(0.01)
2010–16	0.02	0.04	0.00	0.75	0.19	0.25	0.93
	(0.03)	(0.01)	(0.08)	(0.08)	(0.07)	(0.07)	(0.01)

(b) Rolling-window SPX results

	GTARCH						EWMA
	μ	ω	α	β	γ	δ	λ
Mean	0.00	0.02	0.00	0.84	0.14	0.16	0.93
SD	0.00	0.00	0.00	0.02	0.01	0.03	0.004
Min	0.00	0.02	0.00	0.80	0.12	0.09	0.92
Max	0.01	0.03	0.00	0.89	0.17	0.23	0.94

Part (a) shows the results of estimation of GTARCH and EWMA models for SPX data for three subsamples: October 8, 2002 to December 31, 2007 (1278 observations), October 8, 2002 to December 31, 2009 (1772 observations) and January 1, 2010 to December 30, 2016 (1773 observations). Standard deviation (SD) values are given in parentheses for each parameter. Part (b) shows the results of the estimation using rolling windows. Each estimation window size is 2500 observations; we reestimated the models 1000 times. The first window's start date is October 8, 2002 and its end date is December 18, 2012. The start date of the last window is November 14, 2006 and its end date is December 30, 2016.

decreases from 0.96 to 0.93 in later periods. This could indicate a more reactive response to the news (more procyclical) forecast during and after the crisis. Likewise, the asymmetric terms in the GTARCH model (γ and δ) increase in later periods, showing a higher degree of risk aversion and procyclicality.

Table 3(b) presents the summary statistics (mean, standard deviation (SD), minimum and maximum) for the rolling-window results. Each estimation window size is 2500 observations; we reestimated models 1000 times. We can see that mean rolling window results are close to the whole sample results reported previously.

Table 4 shows the degree of risk aversion in each model, measured by the correlation between returns r_{t-1} and the log difference of fitted conditional variance $\log(\sigma_t^2/\sigma_{t-1}^2)$ for each model. The more negative correlation implies a higher degree of risk aversion because of the asymmetrically higher volatility for negative returns. The table shows that the highest degree of risk aversion is captured by the GTARCH

TABLE 4 Degree of risk aversion: SPX and TSX.

(a) SPX

	GTARCH	GTARCH0	GJR-GARCH	GARCH	EWMA	RV
SMacro	–0.726	–0.57	–0.658	–0.158		
Spline	–0.764	–0.6	–0.659	–0.183		
No spline	–0.755	–0.544	–0.659	–0.192	–0.146	–0.111

(b) TSX

	GTARCH	GTARCH0	GJR-GARCH	GARCH	EWMA	RV
SMacro	–0.744	–0.614	–0.628	–0.213		
Spline	–0.762	–0.638	–0.661	–0.245		
No spline	–0.715	–0.584	–0.632	–0.255	–0.190	–0.057

This table illustrates the correlation between returns r_t and the log difference of fitted conditional variance $\log(\sigma_t^2/\sigma_{t-1}^2)$ for each model. The last column presents correlation results for the RV estimate of σ_t. A more negative correlation implies a higher degree of risk aversion in the model.

models and the smallest correlation is for the EWMA, GARCH and RV models, which are symmetrical.

Figures 1 and 2 show the annualized GTARCH volatilities for SPX data, while Figures 3 and 4 present similar graphs for TSX. We can see that the low-frequency component is smooth for both SPX and TSX data in the spline-GTARCH model, and the high-frequency component is close to but generally higher than GTARCH. Once the macroeconomic variables are added, the dynamics of low-frequency volatility become much less smooth. This is due to the reaction to macroeconomic volatility in turbulent times affecting the long-run volatility component. The reaction to negative news is also amplified by the asymmetric effect in the GTARCH model.

Overall, the US and Canadian market volatilities have similar dynamics and peaks; however, the Canadian market has a lower level of volatility. For the low-frequency spline component, the highest level during the financial crisis was 17% for TSX compared with more than 40% for SPX.

Figures 5 and 6 show RV versus GTARCH for SPX and TSX. We observe that RV is more procyclical than GTARCH, as the RV graphs exhibit higher peaks and lower levels in calm periods.

FIGURE 1 High- and low-frequency volatility: spline-GTARCH for SPX.

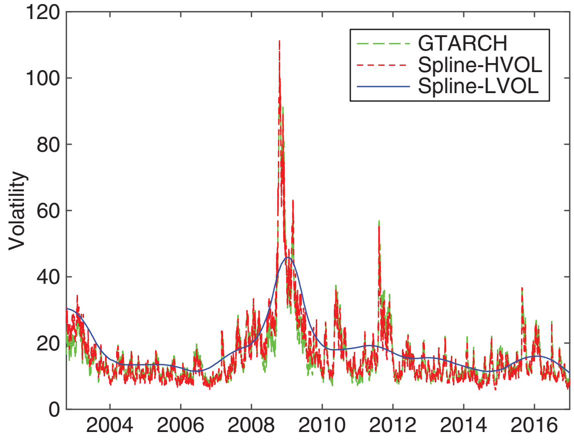
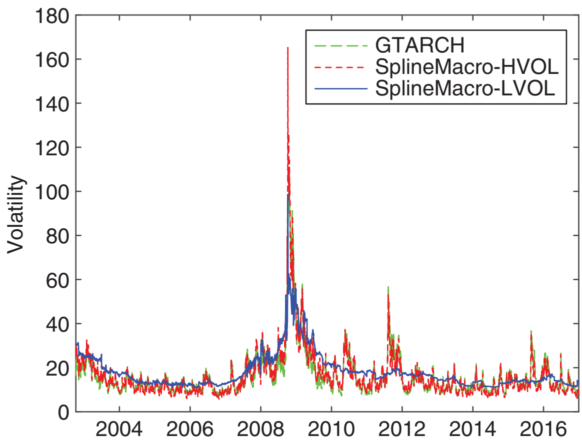


FIGURE 2 High- and low-frequency volatility: spline-macro-GTARCH for SPX.



4 INITIAL MARGIN MEASURES

In this section, we compute the tail risks and perform backtests on all models. Later, we analyze the initial margin models' procyclicality and estimate a three-regime threshold autoregressive model (3TAR) for setting a floor, a ceiling and speed limits on margins.

FIGURE 3 High- and low-frequency volatility: spline-GTARCH for TSX.

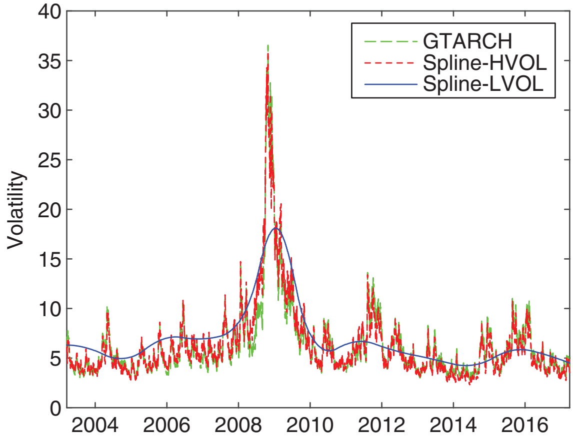
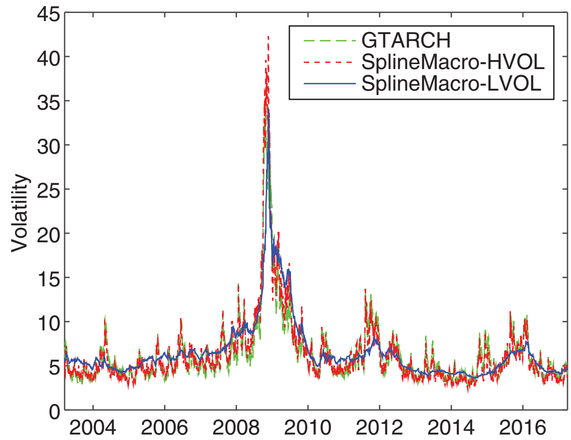


FIGURE 4 High- and low-frequency volatility: spline-macro-GTARCH for TSX.



4.1 Properties of tail risks for setting margin requirements

Figures 7 and 8 show the logarithmic returns and negative values of one-day 99% VaR for SPX and TSX, respectively. We generated one-day 99% VaRs using the Hull and White (1998) bootstrap method and the normal distribution. We used the spline-GTARCH model in these graphs, while all other models are reported in the

FIGURE 5 RV and GTARCH for SPX.

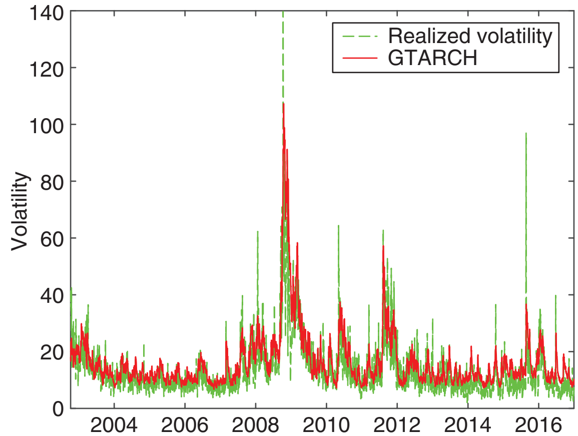
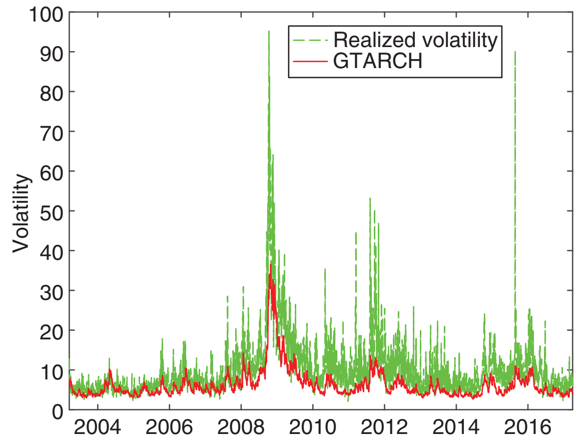


FIGURE 6 RV and GTARCH for TSX.



online appendix (Tables A3 and A4). The margin requirements with the Hull–White method are higher because this method uses the actual returns distribution with fatter tails than the normal distribution.

We show in the online appendix that, while there is no one specific model that always has the highest volatility forecast, those with asymmetric terms (GTARCH, GJR-GARCH and GTARCH0) produce higher forecasts and tail risks than sym-

FIGURE 7 SPX log returns and one-day VaR: spline-GTARCH.

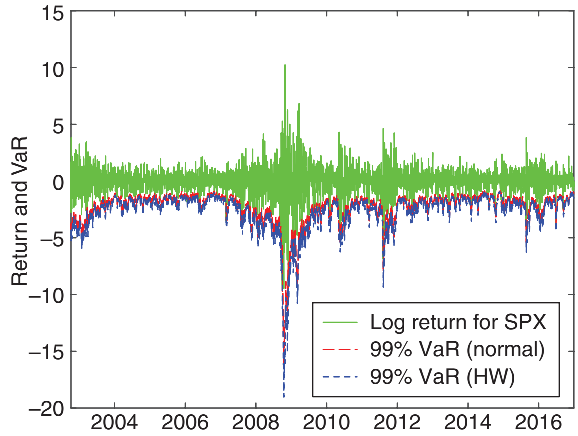
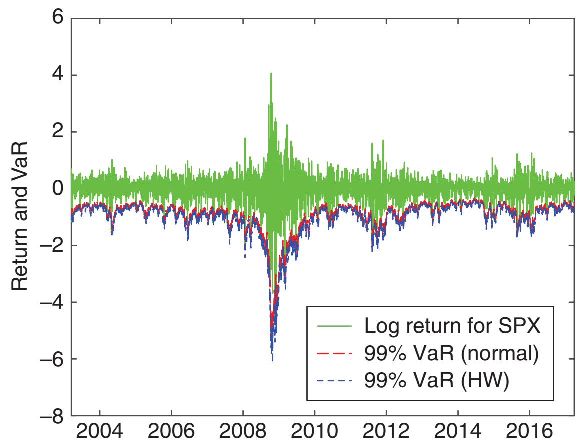


FIGURE 8 TSX log returns and one-day VaR: spline-GTARCH.



metrical GARCH and EWMA models. Thus, models accounting for risk aversion, such as GTARCH, are useful to ensure that volatility is not underestimated and sufficient margin requirements are set. Other results in the online appendix illustrate that models with spline have smaller volatility forecasts than models with spline-macro at times of low volatility. The opposite is true at times of high volatility. Thus, the spline-macro models turn out to be less procyclical. This may be explained by countercyclical monetary policy as well as faster convergence to a less smooth long-run spline macroeconomic component, as was found from lower persistence of the spline-macro models in Tables 1 and 2.

4.2 Model validation

Backtesting is often used in practice for model validation. If the actual breach rate turns out to be too high, the VaR margin model underestimates risk, which creates a loss for the CCP. Alternatively, if the breach rate is too low, the VaR model over-estimates risk and results in unnecessarily high margin charges for the members of the CCP. Thus, margins can be set based on a VaR that has a reasonable number of backtest violations that fall within some confidence interval.

Table 5 presents the results of backtesting, with the number of breaches for the 90%, 95% and 99% VaRs for SPX and TSX produced by each volatility model for the whole sample period. The table shows 95% confidence intervals with lower and upper bounds for the number of permitted breaches obtained using the Kupiec test. We report the results for VaRs obtained using the Hull and White (1998) method. Our results show that all VaRs for the asymmetric volatility models (GTARCH, GJR-GARCH and GTARCH0) with various quantiles ($q = 90\%, 95\%, 99\%$) pass the Kupiec test at a 5% significance level for both SPX and TSX. At the same time, we find that the EWMA model failed the test, underestimating risk for both SPX and TSX for each quantile q. The GARCH model fails the Kupiec test only for SPX data with $q = 90\%$, overestimating risk.

We also performed the conditional coverage backtests by Christoffersen (1998) and found that all models pass the test, but GARCH models fail the independence test at a 5% significance level. However, past research (see, for example, Lopez 1998) showed the low power of all the above backtests. Moreover, backtesting is only concerned with the number of exceptions and their independence. Regulators are also concerned with the magnitude of exceptions (margin shortfall) as well as any excessive procyclicality of VaR models that increase the speed of margin calls in times of crisis.

In addition to backtesting, we ran RV regressions to assess model performance, similar to Corsi (2009) and Bekaert and Hoerova (2014). Table A6 in the online appendix shows one-day-ahead in-sample performance using the regressions of log RV on log variances estimated by each model. We find that the GTARCH model performs better than all other models for SPX using all three statistics: mean squared error (MSE), mean absolute error (MAE) and Mincer–Zarnowitz (MZ) R^2.

TABLE 5 Backtesting for SPX and TSX VaR models.

(a) Breaches allowed at 95% confidence interval

	Lower bound	Upper bound
$\text{VaR}_{q=90\%}$	310	350
$\text{VaR}_{q=95\%}$	146	175
$\text{VaR}_{q=99\%}$	22	35

(b) Breaches for SPX data

Model	% VaR	GTARCH	GTARCH0	GJR-GARCH	GARCH	EWMA
Spline	90	343	336	336	307	
	95	169	166	167	160	
	99	32	33	33	29	
SMacro	90	336	327	326	308	
	95	171	167	166	160	
	99	34	29	32	30	
No spline	90	336	327	326	308	354
	95	171	167	166	160	176
	99	34	29	32	30	36

(b) Breaches for TSX data

Model	% VaR	GTARCH	GTARCH0	GJR-GARCH	GARCH	EWMA
Spline	90	341	328	327	321	
	95	168	163	162	155	
	99	34	32	31	30	
SMacro	90	330	329	332	312	
	95	164	152	162	151	
	99	33	33	33	27	
No spline	90	330	329	332	312	356
	95	164	152	162	151	182
	99	33	33	33	27	36

This table presents the number of backtest breaches for VaR of SPX and TSX produced by each volatility model. VaR was estimated using the Hull and White (1998) method.

The backtesting results and forecast evaluations using MSE, MAE and MZ R^2 statistics reinforce the need to use asymmetric volatility models that capture risk aversion to make sure that margins are set adequately.

4.3 Procyclicality of the CCP's initial margin requirements

This section explores the procyclicality of margin requirements based on previously estimated VaR models and ways to reduce procyclicality.

We suggest imposing both a floor and a ceiling on margins, by using a 3TAR model and expert judgment based on historical margin settings. We consider a time series of the logarithm of VaR, $y_t = \log(\text{VaR})$, with three regimes. The online appendix gives a description of the 3TAR model. In this model, we estimate the two thresholds that separate the regimes and set the thresholds for the floor and the ceiling, thus providing a margin buffer when volatility is low and using the margin buffer when volatility is high. The floor and ceiling are not fixed and move over time, reflecting market conditions based on a rolling-window estimation. As an example, we evaluate the appropriateness of the 25% margin buffer suggested in the literature. We use log(VaR) for estimation of the 3TAR model, since log transformation smooths the peaks. Then we exponentially transform the threshold values and report them for all the volatility models in Table 6. The rolling-window results of using GTARCH and EWMA models for SPX data with an estimation window of 1000 observations are presented in Figure 9. We show moving ceiling, floor and unmitigated VaR. We can see that, while the bounds change over time, reflecting volatility in the estimation window, the spikes in 2009 and 2011 are considerably smoothed by the ceiling. However, the lower threshold corresponds to at least the 25% quantile of the estimation window.[3]

The 3TAR model provides a straightforward method of simultaneously setting floors and ceilings for the initial margin that are stable and not too procyclical: the one-day margins for the whole sample reported in Table 6 are on average bounded between 1.84% and 2.58% for SPX and between 0.77% and 1.01% for TSX. This way, when volatility is low, the margins are fixed at a conservative floor level that corresponds to an additional buffer of about the 29% quantile of the lowest margins for SPX, and at times of market stress they cannot go above the upper threshold. It is an interesting coincidence that the estimated lower threshold for SPX using the EWMA model corresponds to the 25% of observations in the low-volatility regime, as recommended by the EMIR and Murphy *et al* (2016).

For TSX, the margin buffer is higher: 32% of observations on average. However, at times of stress the higher thresholds correspond on average to 38% of the observation points for both SPX and TSX, which may not appear to be too conservative. We could add the actual historical margins set by CCPs at times of stress here, to see if the upper bound would have been higher and would have resulted in a lower percentage of observations for the high-volatility regime. For comparison, we also estimated

[3] Estimation windows with more observations are desirable for estimating more general GTARCH models with more parameters.

TABLE 6 Thresholds for VaR margins: SPX and TSX. [Table continues on next page.]

(a) SPX data

	Model	Threshold (%)		Proportion (%)			Unit root		
		Lower	Upper	Low regime	Median regime	High regime	Low regime	Median regime	High regime
No spline	GTARCH	1.90	2.44	33	25	42	No	No	No
	GTARCH0	1.91	2.95	30	41	28	No	No	No
	GJR-GARCH	1.88	2.39	31	25	44	No	No	No
	GARCH	1.93	2.47	27	29	44	No	No	No
	EWMA	1.82	2.90	25	44	31	No	No	No
	Average	1.89	2.63	29	33	38			
Spline	GTARCH	1.74	2.66	26	39	35	No	No	No
	GTARCH0	1.92	2.47	33	26	41	No	No	No
	GJR-GARCH	1.87	2.47	36	26	38	No	No	No
	GARCH	1.86	2.29	27	25	48	No	No	No
	Average	1.85	2.48	31	29	40			
SMacro	GTARCH	1.74	2.74	28	40	32	No	No	No
	GTARCH0	1.78	2.74	25	41	34	No	No	No
	GJR-GARCH	1.74	2.18	26	25	49	No	No	No
	GARCH	1.85	2.85	27	43	30	No	No	No
	Average	1.78	2.63	26	37	36			
	Overall average	1.84	2.58	29	33	38			
	RV	1.57	2.27	49	25	26	No	No	No

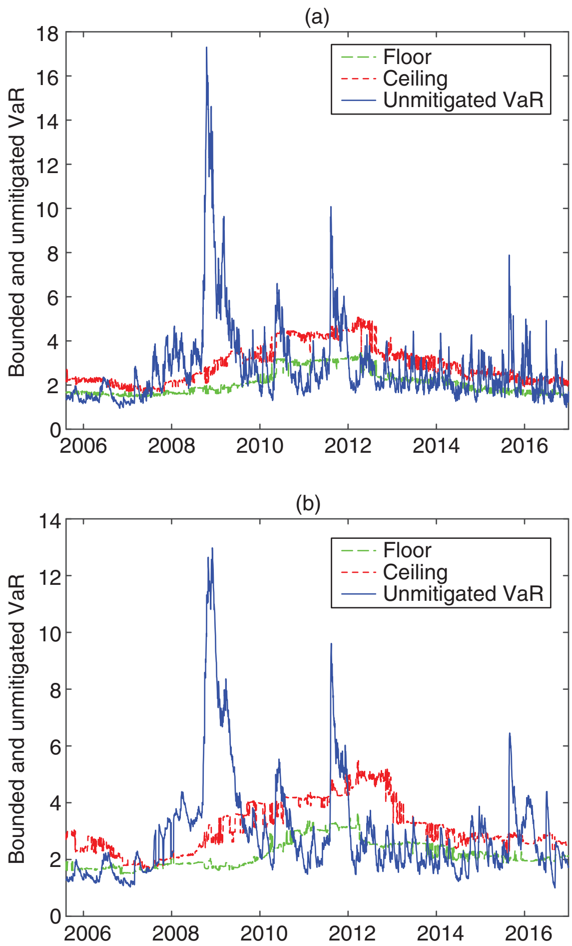
TABLE 6 Continued.

(b) TSX data

	Model	Threshold (%)		Proportion (%)			Unit root		
		Lower	Upper	Low regime	Median regime	High regime	Low regime	Median regime	High regime
No Spline	GTARCH	0.74	0.97	29	29	42	No	No	No
	GTARCH0	0.77	1.10	27	41	32	No	No	No
	GJR-GARCH	0.74	1.01	26	33	40	No	No	No
	GARCH	0.83	1.05	32	29	40	No	No	No
	EWMA	0.79	1.02	34	26	40	No	Yes	Yes
	Average	0.77	1.03	30	32	39			
Spline	GTARCH	0.73	0.94	33	26	41	No	Yes	Yes
	GTARCH0	0.87	1.15	48	25	27	No	Yes	Yes
	GJR-GARCH	0.71	0.93	26	30	44	No	Yes	Yes
	GARCH	0.74	0.95	25	27	48	No	Yes	Yes
	Average	0.76	0.99	33	27	40			
SMacro	GTARCH	0.82	1.12	47	26	27	No	Yes	No
	GTARCH0	0.82	1.04	41	26	33	No	Yes	No
	GJR-GARCH	0.70	0.87	25	27	47	No	No	No
	GARCH	0.75	1.05	26	39	36	No	Yes	No
	Average	0.77	1.02	35	29	36			
	Overall average	0.77	1.01	32	29	38			
	RV	0.52	1.02	25	49	25	No	No	No

This table presents the results of the 3TAR model applied to VaR produced by each volatility model for SPX and TSX. VaR was estimated using the Hull and White (1998) method. In the last row for each data set, we present the results of applying the 3TAR model to VaR using RV.

FIGURE 9 Estimated rolling-window thresholds with three regimes for SPX: (a) GTARCH and (b) EWMA VaR.



a three-regime model for the VaR constructed using the RV model, which resulted in thresholds of 1.57% and 2.27% for SPX and 0.52% and 1.02% for TSX. These thresholds correspond to approximately 25% of observations in the high-volatility regime.

Another method for reducing procyclicality, as suggested in Murphy *et al* (2016), is based on restricting the growth rate of margins. In our analysis, we estimate two speed limits for how quickly margins can be increased and decreased. We estimate the three-regime threshold autoregressive model for the growth rates of margins $y_t =$

$\log(\text{VaR}_t) - \log(\text{VaR}_{t-1})$ for each statistical model. The results presented in Table A6 in the online appendix show that average thresholds for SPX are -4.73% and 0.89%, while those for TSX are -3.88% and 0.55%.

In order to evaluate whether margin floors and ceilings (or upper and lower bounds, for speed limits) would be sufficient in times of crisis, we need to make sure that the time series of VaRs in the high-volatility regime are stationary and revert back inside the bounds. The unit root test results indicate that all models pass the stability test for the SPX, while EWMA and spline models without macroeconomic variables could be unreliable for setting a sustainable ceiling for TSX. If the margins were allowed to be set within two moving bounds and the high-volatility regime were not persistent, margins would be more stable. Such a policy could also be useful to manage expectations at times of stressed liquidity. This model is the limiting case for mitigating procyclicality while sacrificing risk sensitivity.

To evaluate the initial margin models, we minimize a loss function with two competing objectives: risk sensitivity (model accuracy) and mitigation of procyclicality. We note that Wong and Ge (2017) used time series similarity testing against a theoretical "regulatory target model" for measuring the initial margin model performance. We do not use any assumed "correct" model and minimize the loss function while performing a sensitivity analysis of the loss to a different trade-off parameter w, discussed below.

We introduce a parameter $0 \leqslant w \leqslant 1$, which measures the degree of trade-off between the two objectives. The higher the w, the more weight is given to the procyclicality correction, with the limiting case of $w = 1$. If $w = 0$, there is no correction of procyclicality and the whole weight is given to the objective of model accuracy with the most risk-sensitive model. The CCP may have different preferences for w, resulting in more or less model accuracy versus mitigation of procyclicality.

Let the loss function $L(w)$ be defined as a quadratic measure of margin shortfall and procyclicality. We define the overall loss function as the weighted sum of two components:

$$L(w) = (1 - w)L_1 + wL_2. \tag{4.1}$$

Here, L_1 measures quadratic shortfall, while L_2 measures margin variability:

$$L_1 = \frac{1}{T}\sum_{t=t_0}^{T}(r_t + \text{VaR}_t)^2\, I(r_t < -\text{VaR}_t),$$

$$L_2 = \frac{1}{T}\sum_{t=t_0}^{T}\biggl(\text{VaR}_t - \frac{1}{T}\sum_{t=t_0}^{T}\text{VaR}_t\biggr)^2,$$

where r_t is the one-day logarithmic return, VaR_t is a one-day 99% VaR forecasted for time t, T is the number of observations in the sample, t_0 is the first observation for which a VaR forecast is available and $I(\cdot)$ is an indicator function equal to 1 when there is a violation of VaR and 0 otherwise.

Table 7 presents the results of loss function components L_1 and L_2 for each volatility model. For L_1, we present loss, rank and the percentage of VaR violations. For L_2, we present loss and rank. Results for unmitigated VaR (L_1 and L_2), mitigated VaR by ceiling and floor (L_1^{flat} and L_2^{flat}) and mitigated VaR with speed limits (L_1^{growth} and L_2^{growth}) are given.

By construction, the following inequalities hold for every volatility model: $L_1 < L_1^{\text{growth}} < L_1^{\text{flat}}$ and $L_2 > L_2^{\text{growth}} > L_2^{\text{flat}}$. This is because unmitigated VaR has the highest risk sensitivity (the least shortfall) and the highest variability. Setting ceiling and floor values results in less risk sensitivity and more stability. The loss based on speed limits on the VaR is in between, with some smoothness and slightly less variability than unmitigated VaR.

The RV is the most risk-sensitive measure for SPX, and it is ranked as #1 for SPX L_1. However, as we observe RV ex post and do not produce a forecast for it, it is not surprising that it performs the best. For the variability (L_2, L_2^{flat}, L_2^{growth}), the RV, spline-GJR-GARCH and spline-GTARCH0 models are preferable for SPX and TSX.

Next, we compute the overall loss function in (4.1) for $w = \{0, 0.25, 0.5, 0.75, 1\}$. In Table 8, we rank the overall loss function within each group of models: without spline; with spline; and with spline and macroeconomic variables (SMacro). For $w = 0$, there is no procyclicality mitigation, and the overall loss is equal to unmitigated L_1. For $w = 1$, the procyclicality mitigation is the highest, and the overall loss is equal to either L_2^{flat} or L_2^{growth} depending on the mitigation tool used. As we increase the trade-off parameter, the rank might change between unmitigated ($w = 0$) and mitigated margin ($w \geqslant 0.25$), but then the ranks are the same for all weights $w = \{0.25, 0.75, 1\}$. Thus, the exact value of w is not crucial, and any weight for mitigating procyclicality produces a robust model selection.

Overall, with some degree of procyclicality mitigation, the minimum loss is found with the ceiling and floor rather than speed limits. The two best models with the ceiling and floor are RV and GTARCH0 for SPX and spline-GTARCH0 and spline-macro-GTARCH for TSX. Since intraday measures of volatility may not be generally available for some assets, we choose the second best model, which is GTARCH0 (with asymmetry in the GARCH term only). As for TSX, from Table 6 we find that spline models without macroeconomic variables may have unit roots in the high-volatility regime. Thus, we chose a spline-macro-GTARCH model that incorporates asymmetry in both the ARCH and GARCH terms as well as macroeconomic variables. Adding macroeconomic variables helps to reduce model procyclicality for the TSX index. At the same time, simpler asymmetric models with procyclicality correction through ceilings and floors may be sufficient (as in the case of SPX).

The above method is a simple approach to test the sustainability of margin models using a three-regime threshold autoregressive model.

5 CONCLUSIONS AND FURTHER DEVELOPMENT

In this paper we considered asymmetric GARCH models in the threshold GARCH family and proposed a more general spline-GTARCH model that captures high-frequency return volatility and low-frequency macroeconomic volatility as well as an asymmetric response to past negative news in both ARCH and GARCH terms.

We then applied a variety of volatility models, including asymmetric GARCH, GARCH and EWMA, in setting initial margin requirements for CCPs. Since VaR and expected shortfall calculations are typically volatility based, the properties of the underlying volatility models, such as risk aversion, are essential for setting initial margin requirements.

Finally, we showed how to mitigate the procyclicality of initial margins using a three-regime threshold autoregressive model. We set the floor and ceiling on the VaR using estimated thresholds. This model is the limiting case for mitigating procyclicality while sacrificing risk sensitivity. In order to evaluate initial margin models, in addition to backtesting and volatility forecast evaluation, we introduced a loss function with two competing objectives: risk sensitivity and mitigation of procyclicality. The trade-off parameter between these objectives can be selected by the CCP depending on their specific preferences. We found that asymmetric volatility models generally perform better under various trade-off parameters. In future research, more international equity and other assets could be tested.

DECLARATION OF INTEREST

The authors report no conflicts of interest. The authors alone are responsible for the content and writing of the paper. The views expressed in this paper are those of the authors. No responsibility for these views should be attributed to the Bank of Canada.

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TABLE 7 Loss function components of shortfall and variability: SPX and TSX. [Table continues on next three pages.]

(a) SPX data

		Shortfall								
		Unmitigated $L_1 \times 10^4$			Floor and ceiling $L_1^{\text{flat}} \times 10^4$			Speed limits $L_1^{\text{growth}} \times 10^4$		
	Model	**Shortfall**	**Rank**	**% violations**	**Shortfall**	**Rank**	**% violations**	**Shortfall**	**Rank**	**% violations**
No spline	GTARCH	65.6	9	1.0	1073.9	10	2.9	75.4	8	1.4
	GTARCH0	73.6	13	1.0	786.8	1	2.1	71.8	7	1.0
	GJR-GARCH	59.3	5	0.9	1106.1	11	3.0	64.1	4	1.1
	GARCH	71.9	12	0.9	1050.7	9	2.8	81.2	10	1.0
	EWMA	96.2	14	1.0	817.6	2	2.2	107.4	13	1.1
	RV	6.4	1	0.8	1185.7	13	3.4	223.5	14	2.7
Spline	GTARCH	54.8	3	0.9	936.9	6	2.5	63.9	3	1.2
	GTARCH0	59.5	6	0.9	1045.8	7	2.8	47.4	1	0.8
	GJR-GARCH	51.7	2	0.9	1049.7	8	2.9	55.9	2	1.1
	GARCH	63.6	7	0.8	1173.0	12	3.3	69.4	6	0.9
SMacro	GTARCH	65.0	8	1.0	898.1	5	2.3	88.8	11	1.3
	GTARCH0	66.1	10	0.8	893.7	4	2.3	68.9	5	0.8
	GJR-GARCH	55.4	4	0.9	1262.4	14	3.5	80.7	9	1.1
	GARCH	67.1	11	0.9	834.0	3	2.3	94.0	12	0.9

TABLE 7 Continued.

		Variability					
		Unmitigated L_2		Floor and ceiling L_2^{flat}		Speed limits L_2^{growth}	
	Model	**Variability**	**Rank**	**Variability**	**Rank**	**Variability**	**Rank**
No spline	GTARCH	3.004	10	0.357	7	2.743	7
	GTARCH0	2.605	4	0.327	2	2.569	6
	GJR-GARCH	2.704	7	0.389	8	2.552	5
	GARCH	2.214	2	0.354	6	2.111	2
	EWMA	2.873	8	0.439	13	2.808	11
	RV	2.042	1	0.120	1	1.775	1
Spline	GTARCH	3.053	11	0.418	12	2.747	8
	GTARCH0	2.688	6	0.327	3	2.780	9
	GJR-GARCH	2.664	5	0.331	4	2.509	4
	GARCH	2.238	3	0.410	11	2.131	3
SMacro	GTARCH	3.464	13	0.404	10	3.161	13
	GTARCH0	3.466	14	0.397	9	3.578	14
	GJR-GARCH	3.172	12	0.513	14	2.956	12
	GARCH	2.914	9	0.342	5	2.791	10

TABLE 7 Continued.

(b) TSX data

		Shortfall								
		Unmitigated $L_1 \times 10^4$			Floor and ceiling $L_1^{\text{flat}} \times 10^4$			Speed limits $L_1^{\text{growth}} \times 10^4$		
	Model	**Shortfall**	**Rank**	**% violations**	**Shortfall**	**Rank**	**% violations**	**Shortfall**	**Rank**	**% violations**
No spline	GTARCH	5.52	9	1.0	187.4	10	2.8	7.60	11	1.2
	GTARCH0	4.67	7	1.0	155.3	3	2.2	4.64	3	1.0
	GJR-GARCH	4.87	8	0.9	176.0	8	2.5	6.57	9	1.0
	GARCH	3.83	2	0.9	165.1	4	2.3	4.50	2	0.9
	EWMA	5.90	12	1.0	173.1	7	2.5	6.66	10	1.1
	RV	9.02	14	0.9	176.6	9	2.6	19.64	14	1.8
Spline	GTARCH	6.79	13	1.0	196.4	12	2.9	8.93	13	1.3
	GTARCH0	5.58	10	0.9	145.6	1	2.1	5.70	7	0.9
	GJR-GARCH	5.82	11	0.9	197.5	13	2.9	8.40	12	1.0
	GARCH	3.93	3	0.9	193.1	11	2.8	4.49	1	1.0
SMacro	GTARCH	4.64	6	0.9	152.2	2	2.2	5.41	6	1.0
	GTARCH0	4.33	4	0.9	167.8	6	2.3	4.71	5	0.9
	GJR-GARCH	4.59	5	0.9	215.6	14	3.4	6.33	8	1.1
	GARCH	3.67	1	0.8	167.4	5	2.3	4.67	4	0.9

TABLE 7 Continued.

		Variability					
		Unmitigated L_2		Floor and ceiling L_2^{flat}		Speed limits L_2^{growth}	
	Model	**Variability**	**Rank**	**Variability**	**Rank**	**Variability**	**Rank**
No spline	GTARCH	0.423	3	0.052	7	0.393	3
	GTARCH0	0.455	8	0.049	4	0.457	8
	GJR-GARCH	0.415	2	0.057	9	0.394	4
	GARCH	0.447	7	0.052	6	0.426	6
	EWMA	0.511	12	0.055	8	0.494	12
	RV	0.426	4	0.061	11	0.363	1
Spline	GTARCH	0.445	6	0.052	5	0.407	5
	GTARCH0	0.438	5	0.024	1	0.440	7
	GJR-GARCH	0.405	1	0.064	12	0.386	2
	GARCH	0.489	9	0.075	14	0.466	9
SMacro	GTARCH	0.501	10	0.030	2	0.494	11
	GTARCH0	0.547	14	0.035	3	0.553	14
	GJR-GARCH	0.505	11	0.072	13	0.483	10
	GARCH	0.528	13	0.058	10	0.507	13

This table presents the results of loss function components L_1 and L_2 for each volatility model. For L_1, we present loss, rank and the percentage of VaR violations. For L_2, we present loss and rank. Results for unmitigated VaR (L_1 and L_2), mitigated VaR by ceiling and floor (L_1^{flat} and L_2^{flat}) and mitigated VaR with speed limits (L_1^{growth} and L_2^{growth}) are given.

TABLE 8 Overall loss function group ranks: SPX and TSX. [Table continues on next page.]

(a) SPX data

		Unmitigated	Floor and ceiling				Speed limits			
	Model	$w = 0$	$w = 0.25$	$w = 0.50$	$w = 0.75$	$w = 1$	$w = 0.25$	$w = 0.50$	$w = 0.75$	$w = 1$
No spline	GTARCH	3	4	4	4	4	5	5	5	5
	GTARCH0	5	2	2	2	2	4	4	4	4
	GJR-GARCH	2	6	6	6	6	3	3	3	3
	GARCH	4	3	3	3	3	2	2	2	2
	EWMA	6	5	5	5	5	6	6	6	6
	RV	1	1	1	1	1	1	1	1	1
Spline	GTARCH	2	3	3	3	3	3	3	3	3
	GTARCH0	3	1	1	1	1	4	4	4	4
	GJR-GARCH	1	2	2	2	2	2	2	2	2
	GARCH	4	4	4	4	4	1	1	1	1
SMacro	GTARCH	2	3	3	3	3	3	3	3	3
	GTARCH0	3	2	2	2	2	4	4	4	4
	GJR-GARCH	1	4	4	4	4	2	2	2	2
	GARCH	4	1	1	1	1	1	1	1	1

TABLE 8 Continued.

(b) TSX data

		Unmitigated	Floor and ceiling				Speed limits			
	Model	$w = 0$	$w = 0.25$	$w = 0.50$	$w = 0.75$	$w = 1$	$w = 0.25$	$w = 0.50$	$w = 0.75$	$w = 1$
No spline	GTARCH	4	4	4	4	4	3	3	3	3
	GTARCH0	2	1	1	1	1	5	5	5	5
	GJR-GARCH	3	5	5	5	5	2	2	2	2
	GARCH	1	2	2	2	2	4	4	4	4
	EWMA	5	3	3	3	3	6	6	6	6
	RV	6	6	6	6	6	1	1	1	1
Spline	GTARCH	4	2	2	2	2	2	2	2	2
	GTARCH0	2	1	1	1	1	3	3	3	3
	GJR-GARCH	3	3	3	3	3	1	1	1	1
	GARCH	1	4	4	4	4	4	4	4	4
SMacro	GTARCH	4	1	1	1	1	2	2	2	2
	GTARCH0	2	2	2	2	2	4	4	4	4
	GJR-GARCH	3	4	4	4	4	1	1	1	1
	GARCH	1	3	3	3	3	3	3	3	3

This table presents results of loss functions for unmitigated margin, $L(w = 0)$; margin bounded by floor and ceiling, $L^{\text{flat}}(0.25 \leqslant w \leqslant 1)$; and margin bounded by speed limits, $L^{\text{growth}}(0.25 \leqslant w \leqslant 1)$ for SPX and TSX using various trade-off parameters w ranging from 0 to 1. Ranks for each model group are presented.

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